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Research Article

New Optical Soliton Solutions to the Fractional Hyperbolic Nonlinear Schrödinger Equation

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This paper is aimed at investigating the soliton solutions of the hyperbolic nonlinear Schrödinger equation. Exact analytical solutions of the model are acquired through applying an integration method, namely, the Sine-Gordon method. It is observed that the method is able to efficiently determine the exact solutions for this equation. Graphical simulations corresponding to some of the results obtained in the paper are also drawn. These results can help us better understand the behavior and performance of this model. The procedure implemented in this paper can be recommended in solving other equations in the field. All calculations and graphing are performed using powerful symbolic computational packages in Mathematica software.

1. Introduction

Finding exact solutions for differential equations, including ordinary or partial derivatives, is always an important challenge in mathematics, physics, and engineering. This process is very difficult or even impossible for some of these equations. Therefore, any method that helps us determine these solutions is of great importance and use. Exact solutions can be used to illustrate many nonlinear phenomena observed in mathematical physics. One of the most appropriate tools for describing many events in nature is to employ differential equations. This importance has made the traces to such equations tangible in many branches of science, including mathematics, physics [1–3], electrical engineering, astronomy, mechanics, economics, and many other existing disciplines [4–6]. Based on these remarkable effects, several analytical methods have been successfully applied to obtain exact solutions of such equations. Some of these methods are the homotopy analysis method [7], the variational iteration method [8], the exp-function method [9], the logistic function method [10], the generalized G'/G-expansion [11], the elliptic finder method [12–14], the exponential rational function idea [15], the modified Kudryashov technique [16], and the subequation method [17]. To see more methods, please refer to [18–20], including, biology, nonlinear optics, economy, and applied science [1, 20–34]. In this article, the authors study the HNSE, which is given in the form [35]:

$$iD_y^{\alpha}u + \frac{1}{2}\left(D_x^{2\alpha} - D_t^{2\alpha}\right)u + |u|^2 u = 0, \quad 0 < \alpha \leq 1. \tag{1}$$

It is notable that this equation encompasses a wide range of well-known equations through some specific selection of parameters. So far, a variety of techniques have been used successfully to find the exact solutions to the HNS equation (1). This article contains the following sections. A brief mathematical description of the conformable derivative used in this paper is provided in the second section of this paper. Then, the method used is introduced in the third section. The fourth section involved the exact solutions obtained by employing the analytical method equation and graphical behavior are discovered. Finally, conclusions are presented in the last section of the article.

2. The Conformable Derivative

Biswas proposed an interesting definition of derivative called conformable derivative [1]. This derivative can be considered to be a natural extension of the classical derivative. Furthermore, conformable derivative satisfies all the properties of the standard calculus, for instance, the chain rule.

Definition 1. Let $f:[0,\infty)\longrightarrow\mathbb{R}$, the conformable derivative of a function $f(t)$ of order α, is defined as

$$D_t^\alpha f(t)=\lim_{\epsilon\longrightarrow 0}\frac{f\left(t+\epsilon t^{1-\alpha}\right)-f(t)}{\epsilon},\quad \alpha\in(0,1],t>0. \tag{2}$$

This new definition satisfies the following properties.

Definition 2. Suppose that $c\geq 0$ and $t\geq c$, let h be a function defined on c, t as well as $\alpha\in R$. Then, the α-fractional integral of h is given by

$$_tI_c^\alpha h(t)=\int_c^\alpha \frac{h(x)}{x^{1-\alpha}}dx, \tag{3}$$

if the Riemann improper integral exists.

Theorem 3. *Let $\alpha\in(0,1]$, f, g be α-differentiable at a point t, then*

$$\begin{aligned} D_t^\alpha(af+bg)&=aD_t^\alpha(f)+bD_t^\alpha(g), \text{for } a,b\in\mathbb{R},\\ D_t^\alpha(t^\mu)&=\mu t^{\mu-\alpha}, \text{for } \mu\in\mathbb{R},\\ D_t^\alpha(fg)&=fD_t^\alpha(g)+gD_t^\alpha(f),\\ D_t^\alpha\left(\frac{g}{g}\right)&=\frac{gD_t^\alpha(f)-fD_t^\alpha(g)}{g^2}. \end{aligned} \tag{4}$$

Theorem 4. *Let h be a differentiable function and _ is the order of the conformable derivative. Let g be a differentiable function defined in the range of h, then*

$$D_t^\alpha(fog)(t)=t^{1-\alpha}g(t)^{\alpha-1}g'(t)D_t^\alpha(f(t))_{t=g(t)}, \tag{5}$$

where "prime" is the classical derivative with respect to t.

3. Structure of the Sine-Gordon Method

In order, we consider the Sine-Gordon equation as follows:

$$\psi_{xyt}=\alpha\sin(\psi); \tag{6}$$

here, α is a nonzero constant. We exert the change

$$\psi(x,y,t)=U(\xi),\quad \xi=\eta(x+y+vt); \tag{7}$$

here, v is the traveling wave velocity. Replace Equation (8) in Equation (7)

$$U''=\frac{\alpha}{v\mu^2}\sin(u(\xi)). \tag{8}$$

By simplifying Equation (8), we have

$$\left[\left(\frac{U}{2}\right)'\right]^2=\frac{\alpha}{v\mu^2}\sin^2\left(\frac{U(\xi)}{2}\right)+C. \tag{9}$$

In Equation (9), C is the integration constant. We suppose $C=0$, $w(\xi)=U(\xi)/2$, and $f^2=\alpha/v\mu^2$, so Equation (9) detracts to

$$w'(\xi)^2=f^2\sin^2(w(\xi)). \tag{10}$$

In simple terms, we have

$$w'(\xi)=f\sin(w(\xi)). \tag{11}$$

Inserting $f=1$, we have

$$w'(\xi)=\sin(w(\xi)). \tag{12}$$

We have solutions of Equation (12) as follows:

$$\begin{aligned} &\sin(w(\xi))=\operatorname{sech}(\xi) \text{ or } \cos(w(\xi))=\tanh(\xi),\\ &\sin(w(\xi))=\operatorname{icsc}h(\xi) \text{ or } \cos(w(\xi))=\coth(\xi). \end{aligned} \tag{13}$$

For constructing the solutions of NLPDE as follows:

$$N\left(\psi,\psi_t,\psi_x,\psi_y,\psi_{tt},\ldots\cdots\right)=0. \tag{14}$$

Using the following variation:

$$U(w)=\sum_{j=1}^n \cos^{j-1}(w)\times\left[B_j\sin(w)+A_j\cos(w)\right]+A_0, \tag{15}$$

by using Equation (13), we have the solution of Equation (15) as follows:

$$\begin{aligned} U_1(\xi)&=\sum_{j=1}^n \tanh^{j-1}(\xi)\times\left[B_j\operatorname{sech}(\xi)+A_j\tanh(\xi)\right]+A_0,\\ U_2(\xi)&=\sum_{j=1}^n \coth^{j-1}(\xi)\times\left[B_j\operatorname{csch}(\xi)+A_j\coth(\xi)\right]+A_0. \end{aligned} \tag{16}$$

We obtain n by balancing in [10]. Then, by substituting Equation (15) into ODE concluded from Equation (14), we have a system of algebraic equations of $\sin^i(\xi)$ and $\cos^i(\xi)$. Then, by equating of coefficients, we obtain the necessary coefficients. By substituting these coefficients in (15), we extract the solutions of Equation (14).

4. Solution Procedure

To determine the solitary solution of Equation (1), we first define the following new variables:

$$u(x, y, t) = \hbar(\xi)e^{i\theta},\quad \xi = \left(\frac{1}{\alpha}\right)x^{\alpha} + \left(\frac{\mu}{\alpha}\right)y^{\alpha} - \left(\frac{\sigma}{\alpha}\right)t^{\alpha},\quad \theta = \left(\frac{a}{\alpha}\right)x^{\alpha} + \left(\frac{b}{\alpha}\right)y^{\alpha} + \left(\frac{d}{\alpha}\right)t^{\alpha} + \theta_0. \tag{17}$$

Substituting Equation (2) in Equation (1) and comparing real and imaginary parts, respectively, one can obtain

$$\left(a^2 + 2b - d^2\right)\hbar - 2\hbar^3 + \left(\sigma^2 - 1\right)\hbar'' = 0,\quad \mu = -(a + d\sigma). \tag{18}$$

Taking balance principles between $\hbar''$ and $\hbar^3$ into account in Equation (10) yields $m = 1$. Immediately, the general structure for the solution to the problem, which is presented in (7), is determined as follows:

$$\hbar(\xi) = B_1 \sin(\xi) + A_1 \cos(\xi) + A_0. \tag{19}$$

Following the steps mentioned for the method by substituting Equation (15) along with Equation (8) into Equation (10), we get a polynomial in $\sin(\xi)$, $\cos(\xi)$. Equating the coefficient of same power of $\sin^i(\xi)$, $\cos^i(\xi)(i = 0, 1, 2, \cdots)$, we obtain the system of algebraic equations, and by solving this system, we obtained equations for $A_0, A_1, B_1, a, b, d, \mu$, and σ. Now, by solving obtained systems, we get the following values:

Set 1:

$$A_0 = \frac{\sqrt{2a^2 - 2d^2 - 3\sigma^2 + 4b + 3}}{2},\quad A_1 = \frac{\sqrt{2a^2 - 2d^2 + \sigma^2 + 4b - 1}}{4},\quad B_1 = \frac{\sqrt{2a^2 - 2d^2 + \sigma^2 + 4b - 1}}{4}. \tag{20}$$

So, we obtain the following dark optical soliton:

$$\hbar_1(\xi) = \frac{\sqrt{2a^2 - 2d^2 + \sigma^2 + 4b - 1}}{4}\operatorname{sech}(\xi) + \frac{\sqrt{2a^2 - 2d^2 + \sigma^2 + 4b - 1}}{4}\tanh(\xi) + \frac{\sqrt{2a^2 - 2d^2 - 3\sigma^2 + 4b + 3}}{2}. \tag{21}$$

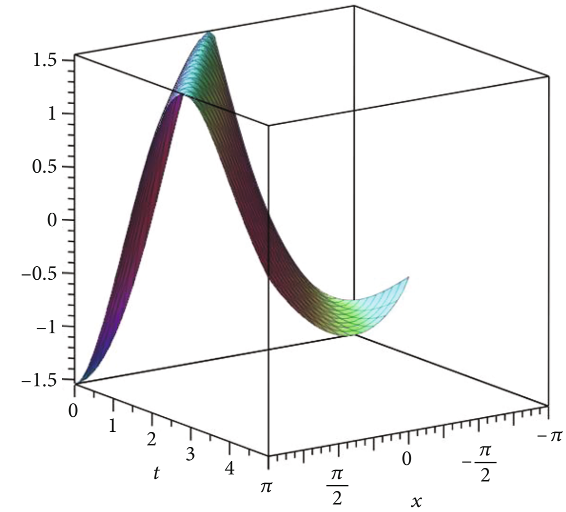

Figure 1: Dynamic behaviors of solution $u_1(x, y, t)$ given by (22) for $t = 0..5$, $x = -\pi..\pi$, for $\alpha = 0.8$.

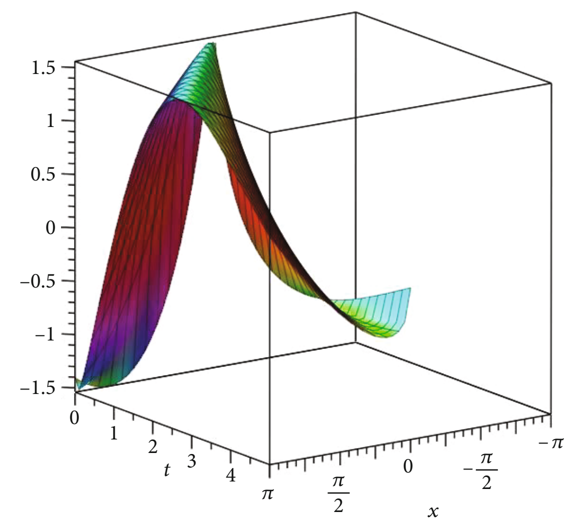

Figure 2: Dynamic behaviors of solution $u_1(x, y, t)$ given by (22) for $t = 0..5$, $x = -\pi..\pi$, for $\alpha = 0.5$.

So we have optical dark soliton solution of (1) as follows:

$$u_1(x, y, t) = \left[\frac{\sqrt{2a^2 - 2d^2 + \sigma^2 + 4b - 1}}{4}\operatorname{sech}\left(\left(\frac{1}{\alpha}\right)x^{\alpha} + \left(\frac{\mu}{\alpha}\right)y^{\alpha} - \left(\frac{\sigma}{\alpha}\right)t^{\alpha}\right) + \frac{\sqrt{2a^2 - 2d^2 + \sigma^2 + 4b - 1}}{4}\tanh\left(\left(\frac{1}{\alpha}\right)x^{\alpha} + \left(\frac{\mu}{\alpha}\right)y^{\alpha} - \left(\frac{\sigma}{\alpha}\right)t^{\alpha}\right) + \frac{\sqrt{2a^2 - 2d^2 - 3\sigma^2 + 4b + 3}}{2}\right]\exp\left(i\left(\left(\frac{a}{\alpha}\right)x^{\alpha} + \left(\frac{b}{\alpha}\right)y^{\alpha} + \left(\frac{d}{\alpha}\right)t^{\alpha} + \theta_0\right)\right). \tag{22}$$

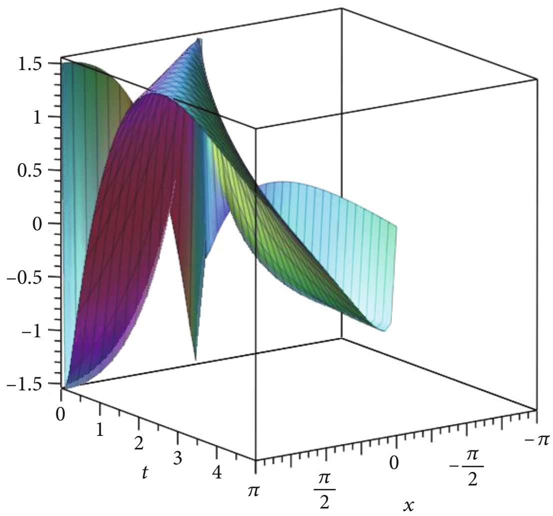

Figure 3: Graphical representation of solution $u_1(x,y,t)$ given by (22) for $t=0..5$, $x=-\pi..\pi$, for $\alpha=0.2$.

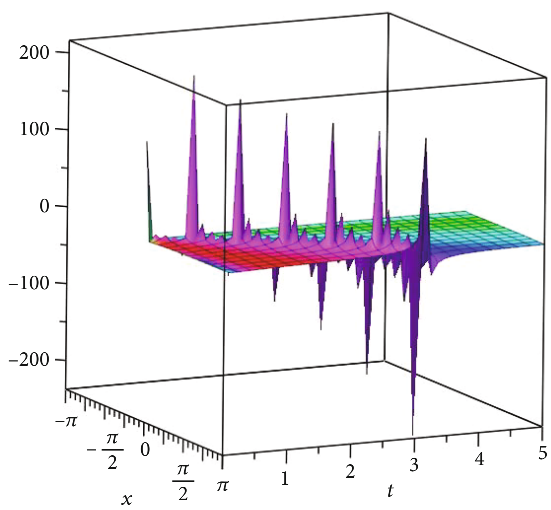

Figure 4: Graphical representation of solution $u_2(x,y,t)$ given by (23) for $t=0..5$, $x=-\pi..\pi$, for $\alpha=0.8$.

And the dark singular soliton is

$$u_2(x,y,t)=\left[\frac{\sqrt{2a^2-2d^2+\sigma^2+4b-1}}{4}\operatorname{csch}\cdot\left(\left(\frac{1}{\alpha}\right)x^\alpha+\left(\frac{\mu}{\alpha}\right)y^\alpha-\left(\frac{\sigma}{\alpha}\right)t^\alpha\right)+\frac{\sqrt{2a^2-2d^2+\sigma^2+4b-1}}{4}\coth\cdot\left(\left(\frac{1}{\alpha}\right)x^\alpha+\left(\frac{\mu}{\alpha}\right)y^\alpha-\left(\frac{\sigma}{\alpha}\right)t^\alpha\right)+\frac{\sqrt{2a^2-2d^2-3\sigma^2+4b+3}}{2}\right]\exp\cdot\left(i\left(\left(\frac{a}{\alpha}\right)x^\alpha+\left(\frac{b}{\alpha}\right)y^\alpha+\left(\frac{d}{\alpha}\right)t^\alpha+\theta_0\right)\right). \tag{23}$$

Set 2:

$$A_0=0,\quad A_1=\frac{\sqrt{3}}{6}\sqrt{6a^2-6d^2-6\sigma^2+12b+6},\quad B_1=\frac{1}{6}\sqrt{6a^2-6d^2-6\sigma^2+12b+6}. \tag{24}$$

The optical dark soliton solution is

$$u_3(x,y,t)=\left[\frac{1}{6}\sqrt{6a^2-6d^2-6\sigma^2+12b+6}\operatorname{sech}\cdot\left(\left(\frac{1}{\alpha}\right)x^\alpha+\left(\frac{\mu}{\alpha}\right)y^\alpha-\left(\frac{\sigma}{\alpha}\right)t^\alpha\right)+\frac{\sqrt{3}}{6}\sqrt{6a^2-6d^2-6\sigma^2+12b+6}\tanh\cdot\left(\left(\frac{1}{\alpha}\right)x^\alpha+\left(\frac{\mu}{\alpha}\right)y^\alpha-\left(\frac{\sigma}{\alpha}\right)t^\alpha\right)\right]\times\exp\left(i\left(\left(\frac{a}{\alpha}\right)x^\alpha+\left(\frac{b}{\alpha}\right)y^\alpha+\left(\frac{d}{\alpha}\right)t^\alpha+\theta_0\right)\right). \tag{25}$$

And dark singular soliton is

$$u_4(x,y,t)=\left[\frac{1}{6}\sqrt{6a^2-6d^2-6\sigma^2+12b+6}\operatorname{csch}\cdot\left(\left(\frac{1}{\alpha}\right)x^\alpha+\left(\frac{\mu}{\alpha}\right)y^\alpha-\left(\frac{\sigma}{\alpha}\right)t^\alpha\right)+\frac{\sqrt{3}}{6}\sqrt{6a^2-6d^2-6\sigma^2+12b+6}\coth\cdot\left(\left(\frac{1}{\alpha}\right)x^\alpha+\left(\frac{\mu}{\alpha}\right)y^\alpha-\left(\frac{\sigma}{\alpha}\right)t^\alpha\right)\right]\times\exp\left(i\left(\left(\frac{a}{\alpha}\right)x^\alpha+\left(\frac{b}{\alpha}\right)y^\alpha+\left(\frac{d}{\alpha}\right)t^\alpha+\theta_0\right)\right). \tag{26}$$

Set 3:

$$A_0=\frac{\sqrt{2a^2-2d^2+4b}}{2},\quad A_1=\frac{\sqrt{2a^2-2d^2+\sigma^2+4b-1}}{4},\quad B_1=0. \tag{27}$$

The optical dark soliton solution is

$$u_5(x,y,t)=\left[\frac{\sqrt{2a^2-2d^2+\sigma^2+4b-1}}{4}\tanh\left(\left(\frac{1}{\alpha}\right)x^\alpha+\left(\frac{\mu}{\alpha}\right)y^\alpha-\left(\frac{\sigma}{\alpha}\right)t^\alpha\right)+\frac{\sqrt{2a^2-2d^2+4b}}{2}\right]\exp\cdot\left(i\left(\left(\frac{a}{\alpha}\right)x^\alpha+\left(\frac{b}{\alpha}\right)y^\alpha+\left(\frac{d}{\alpha}\right)t^\alpha+\theta_0\right)\right). \tag{28}$$

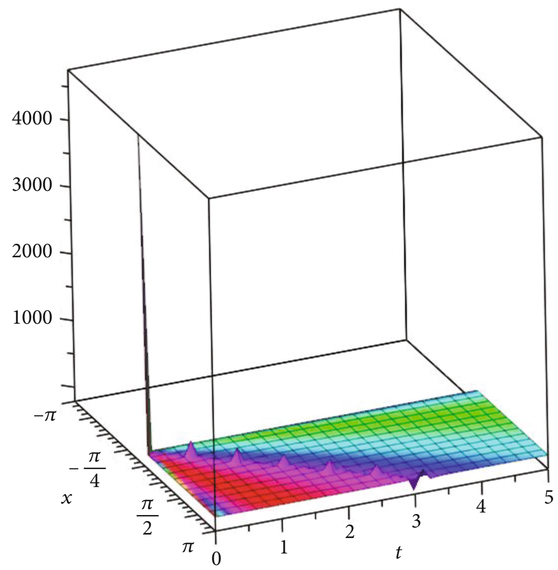

Figure 5: Graphical representation of solution $u_2(x, y, t)$ given by (23) for $t = 0..5, x = -\pi..\pi$, for $\alpha = 0.5$.

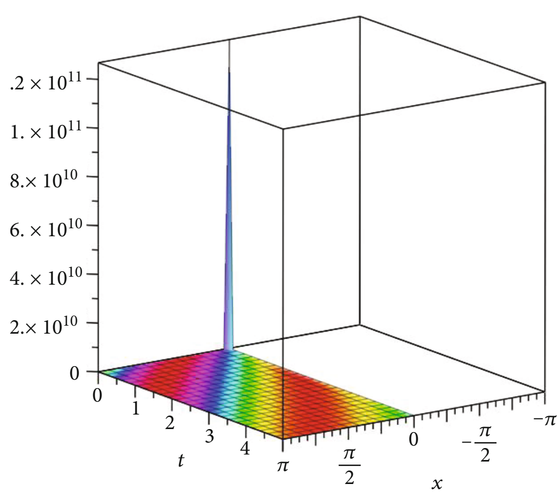

Figure 6: Graphical representation of solution $u_2(x, y, t)$ given by (23) for $t = 0..5, x = -\pi..\pi$, for $\alpha = 0.2$.

And dark singular soliton is

$$u_6(x, y, t) = \left[\frac{\sqrt{2a^2 - 2d^2 + \sigma^2 + 4b - 1}}{4} \coth\left(\left(\frac{1}{\alpha}\right)x^\alpha + \left(\frac{\mu}{\alpha}\right)y^\alpha - \left(\frac{\sigma}{\alpha}\right)t^\alpha\right) + \frac{\sqrt{2a^2 - 2d^2 + 4b}}{2}\right] \exp \cdot \left(i\left(\left(\frac{a}{\alpha}\right)x^\alpha + \left(\frac{b}{\alpha}\right)y^\alpha + \left(\frac{d}{\alpha}\right)t^\alpha + \theta_0\right)\right). \tag{29}$$

In Figures 1–9, we see that the graphs of the answers are very similar and the only difference is in the degree of oscillation of the graph.

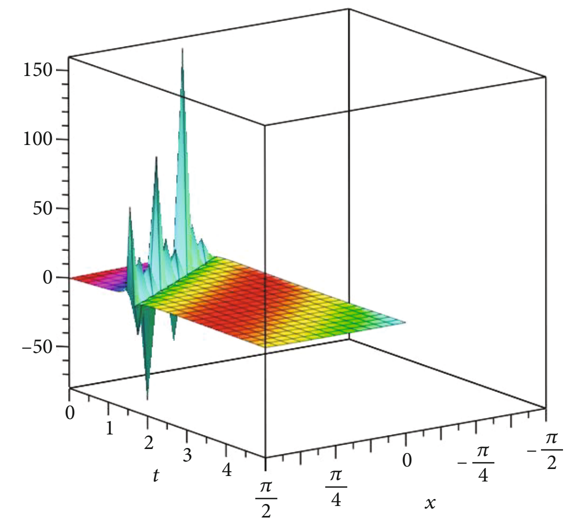

Figure 7: Graphical representation of solution $u_6(x, y, t)$ given by (29) for $t = 0..5, x = -\pi..\pi$, for $\alpha = 0.8$.

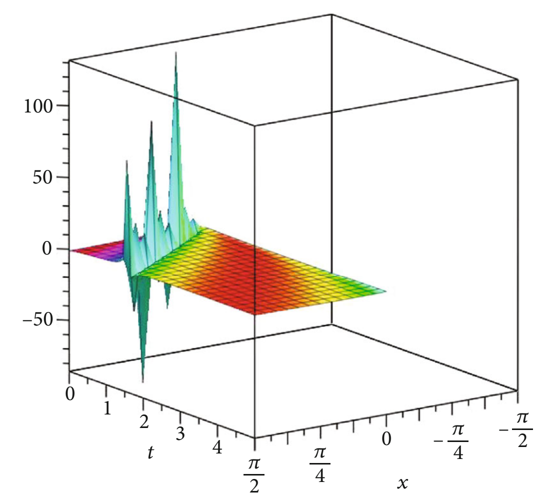

Figure 8: Graphical representation of solution $u_6(x, y, t)$ given by (29) for $t = 0..5, x = -\pi..\pi$, for $\alpha = 0.5$.

5. Concluding Remarks

In this study, some new solitary exact solutions of the hyperbolic Schrödinger equation are obtained with the aid of an efficient analytic method. The structure considered for the equation consists of a series of arbitrary parameters that lead to many well-known models by considering certain options for them. One of the main advantages of this method is the determination of different categories of solutions for the equation in a single framework; this means that the method can determine different types of solutions for the equation in a single process. Furthermore, one can easily deduce that the methods used in this study are very simple but very efficient methodologies for solving NPDEs. We have performed all necessary calculations for obtaining and plotting Figures 1–9 through the implementation of the symbolic computations in Mathematica software.

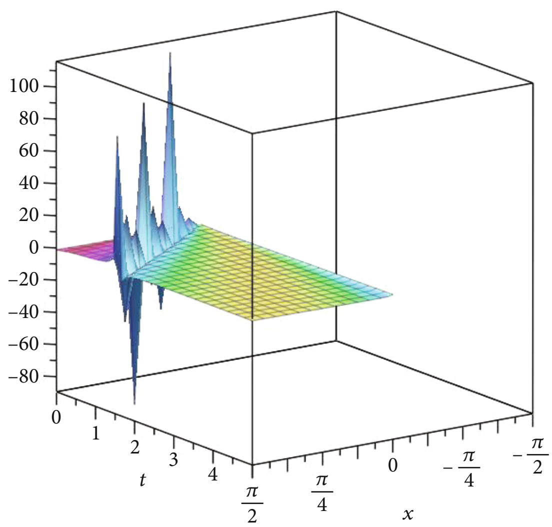

FIGURE 9: Graphical representation of solution $u_6(x, y, t)$ given by (29) for $t = 0..5, x = -\pi..\pi$, for $\alpha = 0.2$.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares no conflicts of interest.

References

[1] A. Biswas, "Chirp-free bright optical soliton perturbation with Fokas-Lenells equation by traveling wave hypothesis and semi-inverse variational principle," *Optik*, vol. 170, pp. 431–435, 2018.

[2] A. Biswas, M. Ekici, A. Sonmezoglu, and R. T. Alqahtani, "Optical soliton perturbation with full nonlinearity for Fokas-Lenells equation," *Optik*, vol. 165, pp. 29–34, 2018.

[3] A. Biswas and S. Arshed, "Optical solitons in presence of higher order dispersions and absence of self- phase modulation," *Optik*, vol. 174, pp. 452–459, 2018.

[4] S. W. McCue, M. El-Hachem, and M. J. Simpson, "Exact sharp-fronted travelling wave solutions of the Fisher-KPP equation," *Applied Mathematics Letters*, vol. 114, p. 106918, 2021.

[5] A. Neirameh, "Nonlinear evolution equations and their analytical and numerical solutions," *Advances in Mathematical Physics*, vol. 2021, Article ID 5538516, 2021.

[6] A. Kurt, H. Rezazadeh, M. Senol, A. Neirameh, O. Tasbozan, and M. Eslami, "Two effective approaches for solving fractional generalized Hirota-Satsuma coupled KdV system arising in interaction of long waves," *Journal of Ocean Engineering and Science*, vol. 4, no. 1, pp. 24–32, 2019.

[7] S. J. Liano, *The proposed homotophy analysis technique for the solution of nonlinear problems*, (Ph.D. thesis), Shanghai Jiao Tong University, 1992.

[8] A. M. Wazwaz, "The variational iteration method for solving linear and nonlinear systems of PDEs," *Computers & Mathematics with Applications*, vol. 54, no. 7-8, pp. 895–902, 2007.

[9] J. H. He and X. H. Wu, "Exp-function method for nonlinear wave equations," *Chaos, Solitons and Fractals*, vol. 30, no. 3, pp. 700–708, 2006.

[10] N. A. Kudryashov and P. N. Ryabov, "Analytical and numerical solutions of the generalized dispersive Swift- Hohenberg equation," *Mathematics of Computation*, vol. 286, pp. 171–177, 2016.

[11] J. Manafian, M. Fazli Aghdaei, M. Khalilian, and R. S. Jeddi, "Application of the generalized G'/G-expansion method for nonlinear PDEs to obtaining soliton wave solution," *Optik*, vol. 135, pp. 395–406, 2017.

[12] A. Biswas, M. Ekici, A. Sonmezoglu, A. S. Alshomrani, and M. R. Belic, "Optical solitons with Kudryashov's equation by extended trial function," *Optik*, vol. 202, p. 163290, 2020.

[13] N. Sajid and G. Akram, "Novel solutions of Biswas-Arshed equation by newlyϕ^6-model expansion method," *Optik*, vol. 211, article 164564, 2020.

[14] B. Ghanbari, S. Kumar, M. Niwas, and D. Baleanu, "The Lie symmetry analysis and exact Jacobi elliptic solutions for the Kawahara-KdV type equations," *Results in Physics*, vol. 23, p. 104006, 2021.

[15] K. Hosseini, Z. Ayati, and R. Ansari, "New exact traveling wave solutions of the Tzitzeica type equations using a novel exponential rational function method," *Optik*, vol. 148, pp. 85–89, 2017.

[16] S. M. Ege and E. Misirli, "The modified Kudryashov method for solving some fractional-order nonlinear equations," *Advances in Difference Equations*, vol. 2014, no. 1, 3 pages, 2014.

[17] H. Rezazadeh, A. Neirameh, M. Eslami, A. Bekir, and A. Korkmaz, "A sub-equation method for solving the cubic–quartic NLSE with the Kerr law nonlinearity," *Modern Physics Letters B*, vol. 33, no. 18, article 1950197, 2019.

[18] H. Rezazadeh, M. Mirzazadeh, S. M. Mirhosseini-Alizamini, and A. Neirameh, "Optical solitons of Lakshmanan-Porsezian-Daniel model with a couple of nonlinearities," *Optik*, vol. 164, pp. 414–423, 2018.

[19] H. Rezazadeh, D. Kumar, A. Neirameh, M. Eslami, and M. Mirzazadeh, "Applications of three methods for obtaining optical soliton solutions for the Lakshmanan-Porsezia-Daniel model with Kerr law nonlinearity," *Pramana*, vol. 94, no. 1, pp. 1–11, 2020.

[20] A. Biswas, M. Asma, P. Guggilla et al., "Optical soliton perturbation with Kudryashov's equation by semi-inverse variational principle," *Physics Letters A*, vol. 384, no. 33, p. 126830, 2020.

[21] Y. Shi, M. Pana, and D. Peng, "Replicator dynamics and evolutionary game of social tolerance: the role of neutral agents," *Economics Letters*, vol. 159, pp. 10–14, 2017.

[22] A. Biswas, M. Ekici, A. Sonmezoglu et al., "Chirped optical solitons of Chen-Lee-Liu equation by extended trial equation scheme," *Optik*, vol. 156, pp. 999–1006, 2018.

[23] A. Biswas, Y. Yildrim, E. Yasar, Q. Zhou, S. P. Moshokoa, and M. Belic, "Sub pico-second pulses in mono-mode optical fibers with Kaup-Newell equation by a couple of integration schemes," *Optik*, vol. 167, pp. 121–128, 2018.

[24] A. Biswas, M. Ekici, A. Sonmezoglu, and R. T. Alqahtani, "Sub-pico-second chirped optical solitons in mono-mode fibers with Kaup-Newell equation by extended trial function method," *Optik*, vol. 168, pp. 208–216, 2018.

[25] A. Biswas, "Stochastic perturbation of optical solitons in Schrodinger-Hirota equation," *Optics Communication*, vol. 239, no. 4–6, pp. 461–466, 2004.

[26] S. U. Rehman, A. R. Seadawy, M. Younis, S. T. R. Rizvi, T. A. Sulaiman, and A. Yusuf, "Computing wave solutions and conservation laws of conformable time-fractional Gardner and Benjamin–Ono equations," *Pramana*, vol. 95, p. 43, 2021.

[27] M. Alquran, I. Jaradat, A. Yusuf, and T. A. Sulaiman, "Heart-cusp and bell-shaped-cusp optical solitons for an extended two-mode version of the complex Hirota model: application in optics," *Optical and Quantum Electronics*, vol. 53, p. 26, 2021.

[28] A. Yusuf, "Symmetry analysis, invariant subspace and conservation laws of the equation for fluid flow in porous media," *International Journal of Geometric Methods in Modern Physics*, vol. 17, no. 12, p. 2050173, 2020, (14 pages).

[29] S. Kumar, A. Kumar, and H. Kharbanda, "Abundant exact closed-form solutions and solitonic structures for the double-chain deoxyribonucleic acid (DNA) model," *Brazilian Journal of Physics volume*, vol. 51, no. 4, pp. 1043–1068, 2021.

[30] S. Kumar, D. Kumar, and A. Kumar, "Lie symmetry analysis for obtaining the abundant exact solutions, optimal system and dynamics of solitons for a higher-dimensional Fokas equation," *Chaos Soliton & Fractals*, vol. 142, no. 110507, p. 110507, 2021.

[31] S. Kumar and D. Kumar, "Lie symmetry analysis and dynamical structures of soliton solutions for the (2 + 1)-dimensional modified CBS equation," *International Journal of Modern Physics B*, vol. 34, no. 25, article 2050221, 2020.

[32] B. Kilic and M. Inc, "Soliton solutions for the Kundu-Eckhaus equation with the aid of unified algebraic and auxiliary equation expansion methods," *Journal of Electromagnetic Waves and Applications*, vol. 30, no. 7, pp. 871–879, 2016.

[33] M. Inc, E. Ates, and F. Tchier, "Optical solitons of the coupled nonlinear Schrödinger's equation with spatiotemporal dispersion," *Nonlinear Dynamics*, vol. 85, no. 2, pp. 1319–1329, 2016.

[34] F. Tchier, E. C. Aslan, and M. Inc, "Optical solitons in parabolic law medium: Jacobi elliptic function solution," *Nonlinear Dynamics*, vol. 85, no. 4, pp. 2577–2582, 2016.

[35] K. L. Khalid, A. M. Wazwaz, M. S. Mehanna, and M. S. Osman, "On short-range pulse propagation described by (2+1)-dimensional Schrödinger's hyperbolic equation in nonlinear optical fibers," *Physica Scripta*, vol. 95, no. 7, article 075203, 2020.