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Research Article

Entanglement Research for the Coupled Superconducting Phase Qubit and a Two-Level System

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Entanglement can exist not only in the microscopic system (e.g., atom, photon, and ion trap) but also in macroscopic systems. According to recent research, entanglement can be achieved and controlled in superconducting devices. The quantum dynamics and entanglement mechanism of the coupled superconducting phase qubit and a two-level system (TLS) were demonstrated when the bipartite system was under microwave driving. Besides, the results reveal that when the system was experiencing decoherence, entanglement (concurrence) of the coupled superconducting phase qubit and TLS would oscillate damply with microwave driving time, even exhibiting concurrence sudden death and revival. The coupling effect of the superconducting qubit and TLS system and the resonant microwave together help to achieve entanglement, while concurrence death and concurrence revival are dependent on the decoherence source and mechanism, for example, the resonant microwave driving time acting on the bipartite coupling system. Furthermore, the simulation results show the entanglement of the coupled qubit and TLS system also depends on the purity of the initial states of the system. The article carried out a numerical simulation on the entanglement of different initial states, and the results showed that the entanglement of the coupled system changes with different initial states. For different initial states, entanglement, sudden death, and rejuvenation are still visible.

1. Introduction

Entanglement is considered as a vital resource to achieve quantum computing and quantum computer [1–5]. In the decades, entanglement between the superconducting qubit and superconducting microwave resonator, etc., has been discussed experimentally [6–10] and theoretically [11–15]. Quantum system was proven to interact with the environment inevitably, and quantum system would undergo damped concurrence oscillation, even concurrence death or concurrence revival [16–22]. One type of defects inside the Josephson junctions has been reported as TLSs, which will degrade the performance of the superconducting devices. Decoherence effect was verified to significantly limit the performance of the superconducting quantum system. Entanglement decay induced by the environment is considered as an important research project for quantum information and quantum cryptography [23–27]. However, it leaves two fundamental questions unanswered: first, what is the generation mechanism of entanglement between the superconducting qubit and the TLS; second, how the environment affects entanglement of the system, thereby leading to concurrence death or concurrence revival.

In the present study, the quantum dynamics and entanglement mechanism of the superconducting phase qubit and TLS system were primarily analyzed. First, splitting occurred in the spectroscopy of the superconducting phase qubit, which proves the coupling effect between the superconducting qubit and the TLS. Second, the occupation probability of the superconducting qubit in four states $(p_{0g}, p_{1g}, p_{0e}, p_{1e})$ and excited state p_1 $(p_1 = p_{1g} + p_{1e})$ was simulated. The occupation probability of the superconducting qubit is prone to Rabi oscillation, and p_1 denotes Rabi beating with microwave driving time. Subsequently,

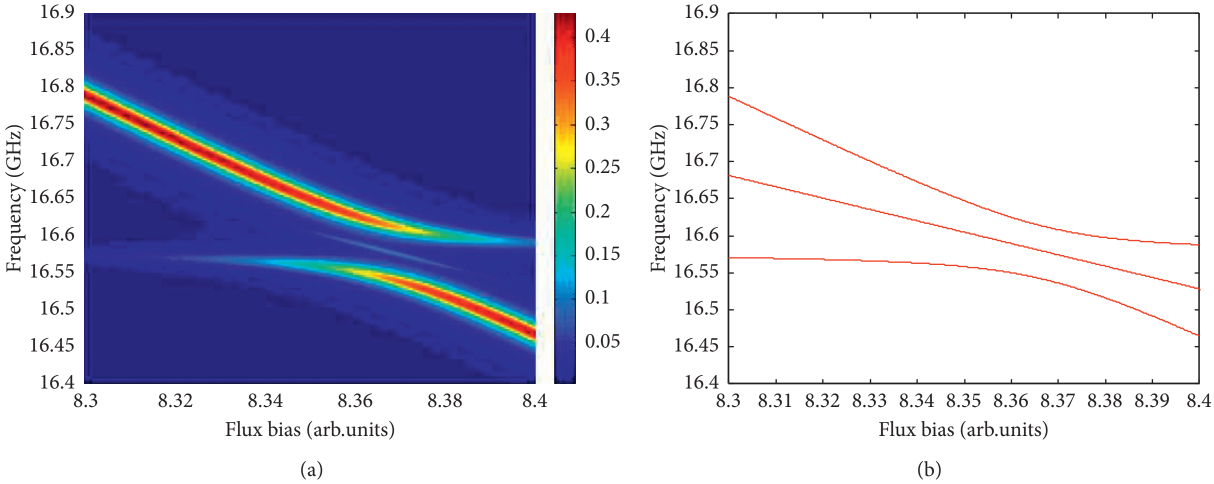


Figure 1: (a) One avoided level crossing near 16.57 GHz is shown in the spectroscopy of the superconducting phase qubit. (b) The simulated excitation spectrum of the superconducting qubit.

entanglement (concurrence) of the coupled system was simulated, suggesting that concurrence oscillates damply with microwave driving time and shows concurrence death and revival phenomena. According to the mentioned results, the superconducting qubit would couple with TLS under the resonant microwave driving, which helps to achieve entanglement, while TLS itself and continuous microwave action are vital for entanglement evolution of the superconducting system. Results indicate that concurrence death and revival were dependent on the resonant microwave driving time acting on the superconducting system. Furthermore, the entanglement of the coupled qubit and TLS system also depends on the purity of the initial states of the system. When the system is prepared in an entangled state, the entanglement differs when the system is initially set in a separated state, which is helpful to the experimental preparation and measurement of quantum entanglement.

2. Quantum Dynamics

Our sample is made of an Nb/AlOX/Nb Josephson junction and measured in a dilution refrigerator (<20 mK, Oxford Triton 400). The proposed bipartite system consists of a superconducting phase qubit and a TLS. The qubit-TLS system would couple with each other when the system was under microwave driving. The Hamiltonian of the proposed system is defined as follows:

$$H = H_{\text{qubit}} + H_{\text{TLS}} + H_I, \tag{1}$$

where H_{qubit} and H_{TLS} denote the Hamiltonian of the superconducting phase qubit and the TLS, respectively. H_I is the coupling effect Hamiltonian of the coupled superconducting phase qubit and TLS. We consider a σ_x-coupling between the superconducting qubit and TLS system, and this method can also be applied to other quantum systems [28].

The simulated spectroscopy and the excitation spectrum of the superconducting phase qubit are shown in Figures 1(a) and 1(b), respectively. Obviously, an avoided level crossing exists in the spectroscopy of the superconducting qubit, proving the coupling effect between the qubit and TLS. The Hamiltonian of the coupled system is written as follows:

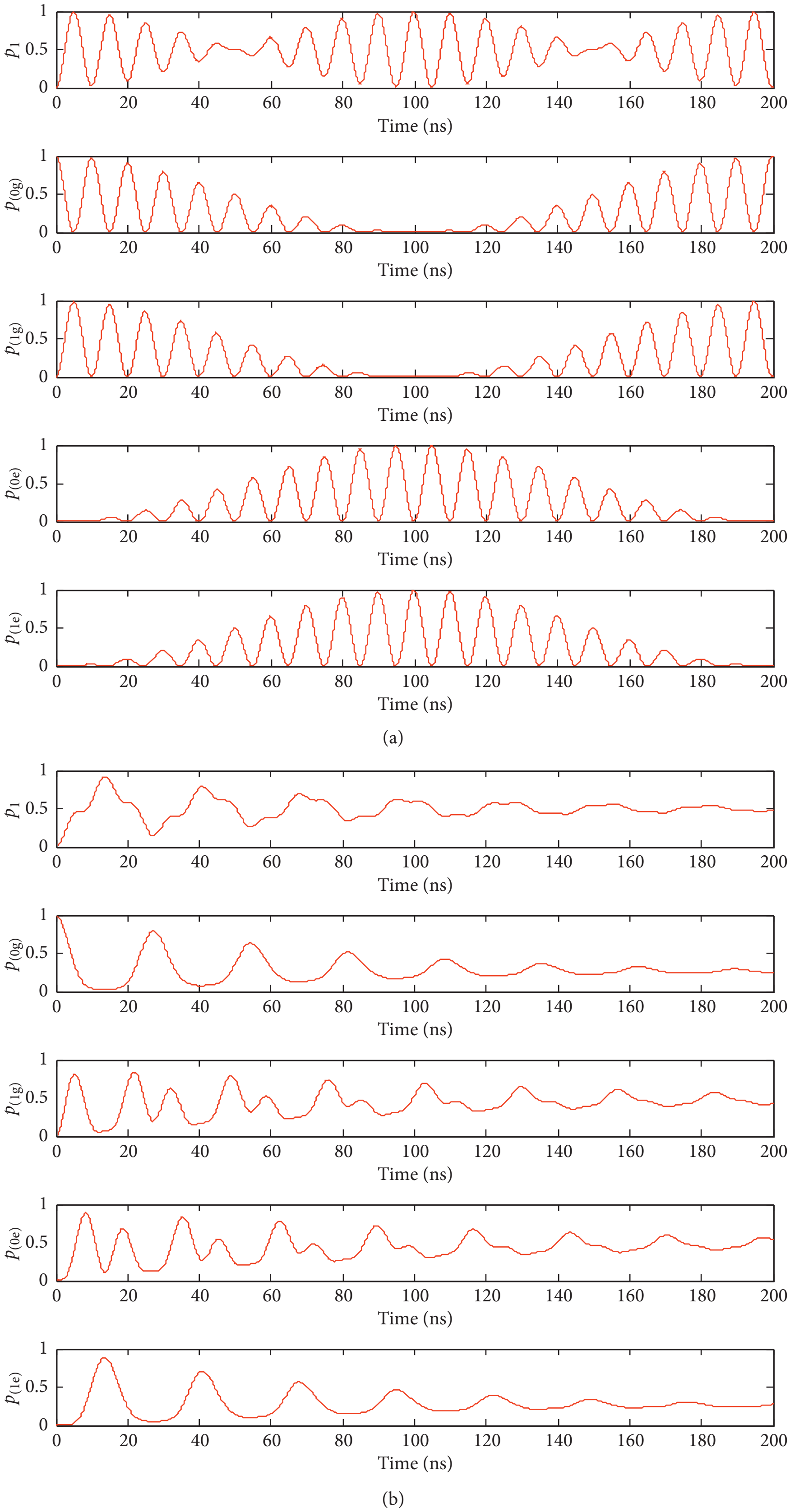
$$H = \begin{pmatrix} 0 & 0 & \eta_{\text{qm}} \sin \omega t & \eta_{qT} \\ 0 & \hbar\omega & \eta_{qT} & \eta_{\text{qm}} \sin \omega t \\ \eta_{\text{qm}} \sin \omega t & \eta_{qT} & \hbar\omega & 0 \\ \eta_{qT} & \eta_{\text{qm}} \sin \omega t & 0 & 2\hbar\omega \end{pmatrix}, \tag{2}$$

where ω denotes the frequency of the microwave field and η_{qT} and η_{qm} are the coupling strength of qubit-TLS and qubit-microwave, respectively. The superconducting qubit was initially prepared in $|0g\rangle$, when under microwave driving, the system would be excited to other three excited states $|1g\rangle$, $|0e\rangle$, and $|1e\rangle$, where the $|0\rangle$ and $|1\rangle$ are the ground and excited states of the superconducting qubit and $|g\rangle$ and $|e\rangle$ are the ground and excited states of the TLS, respectively. The superconducting qubit was initially prepared in its ground state $|0\rangle$, and after a period of microwave driving, the superconducting qubit had a certain occupation probability in state $|1\rangle$. The wave function of the coupled system is expressed as follows:

$$|\psi\rangle = C_{0g}|0g\rangle + C_{1g}|1g\rangle + C_{0e}|0e\rangle + C_{1e}|1e\rangle. \tag{3}$$

In pure situation, using wave function and Schrödinger equation, we have simulated the occupation probability of superconducting qubit in four states $|0g\rangle$, $|1g\rangle$, $|0e\rangle$, and $|1e\rangle$ and excited state $|1\rangle$, $P_1 = |1g\rangle\langle 1g| + |1e\rangle\langle 1e|$. The relevant results are shown in Figure 2(a). It is therefore concluded that the occupation probability of superconducting qubit in four states $|0g\rangle$, $|1g\rangle$, $|0e\rangle$, and $|1e\rangle$ oscillates periodically with Rabi oscillation and P_1 oscillates with Rabi beating periodically.

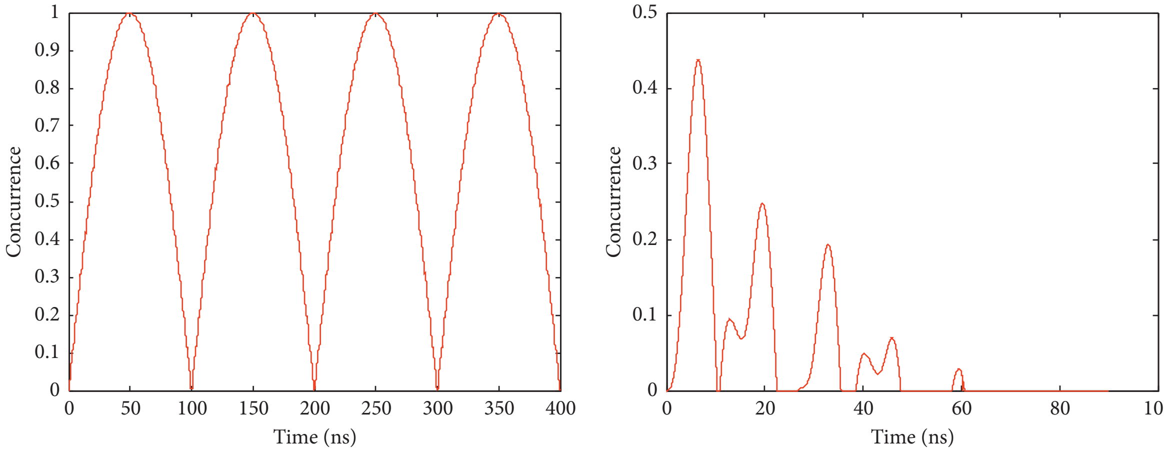
Since superconducting quantum circuits will interact with outside environments inevitably, decoherence effect



(a)

(b)

Figure 2: (a) From top to down. The occupation probability of superconducting qubit in the state $|1\rangle$, $|0g\rangle$, $|1g\rangle$, $|0e\rangle$, and $|1e\rangle$ without decoherence. (b) From top to down. The occupation probability of superconducting qubit in the state $|1\rangle$, $|0g\rangle$, $|1g\rangle$, $|0e\rangle$, and $|1e\rangle$ with decoherence. The coupling strength of qubit-TLS is $\eta_{qT}/\hbar\omega = 0.005$, and the coupling strength of qubit-microwave is $\eta_{\mathrm{qm}}/\hbar\omega = 0.01$. The frequency of the microwave field is $\omega/2\pi = 10\,\mathrm{GHz}$.



(a)

(b)

FIGURE 3: (a) Concurrence of the coupled system in pure situation. (b) Concurrence of the coupled system with decoherence.

should be considered when the quantum system is under external noise interference. In the mixed state situation, density matrix instead of the wave function was employed to discuss the dynamics characteristic of the superconducting quantum system. The density matrix and Lindblad master equation are expressed in equations (4) and (5), respectively, and the second part of equation (5) is the decoherence term:

$$\rho = \begin{pmatrix} \rho_{0g0g} & \rho_{1g0g} & \rho_{0e0g} & \rho_{1e0g} \\ \rho_{0g1g} & \rho_{1g1g} & \rho_{0e1g} & \rho_{1e1g} \\ \rho_{0g0e} & \rho_{1g0e} & \rho_{0e0e} & \rho_{1e0e} \\ \rho_{0g1e} & \rho_{1g1e} & \rho_{0e1e} & \rho_{1e1e} \end{pmatrix}, \tag{4}$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{1}{i\hbar}[H,\rho] + \sum_j \Gamma_j\left(a^- \rho a^+ - \frac{1}{2}a^+ a^- \rho - \frac{1}{2}\rho a^+ a^-\right). \tag{5}$$

The probability for the qubit in state $|1\rangle$ is defined as $P_1 = \rho_{1g1g} + \rho_{1e1e}$. The simulation results are shown in Figure 2(b). The occupation probability of the state $|0g\rangle$, $|1g\rangle$, $|0e\rangle$, and $|1e\rangle$ oscillates damply with microwave duration. If the microwave driving time were sufficiently long, the occupation probability of four states would be close to 0.25, while the occupation probability for the superconducting qubit in the excited state $|1\rangle$ would be nearly 0.5, which is consistent with our previous experimental measurement results [6].

3. Entanglement Mechanism

Entanglement research between the superconducting phase qubit and TLS or superconducting cavity helps to achieve superconducting quantum computing and quantum computers. This study primarily discussed the entanglement (concurrence) mechanism of the coupled superconducting quantum system. In pure situation, the wave function of the system can be expressed in equation (3), and concurrence of the system is defined as follows [29]:

$$C(|\psi\rangle) = 2\left|C_{0g}C_{1e} - C_{1g}C_{0e}\right|. \tag{6}$$

As shown in Figure 3(a), obviously, concurrence between superconducting phase qubit and TLS oscillates periodically with microwave driving time. The maximum entanglement value is 1, and the minimum value is 0.

If decoherence is considered, the concurrence under mixed states is given by [29]

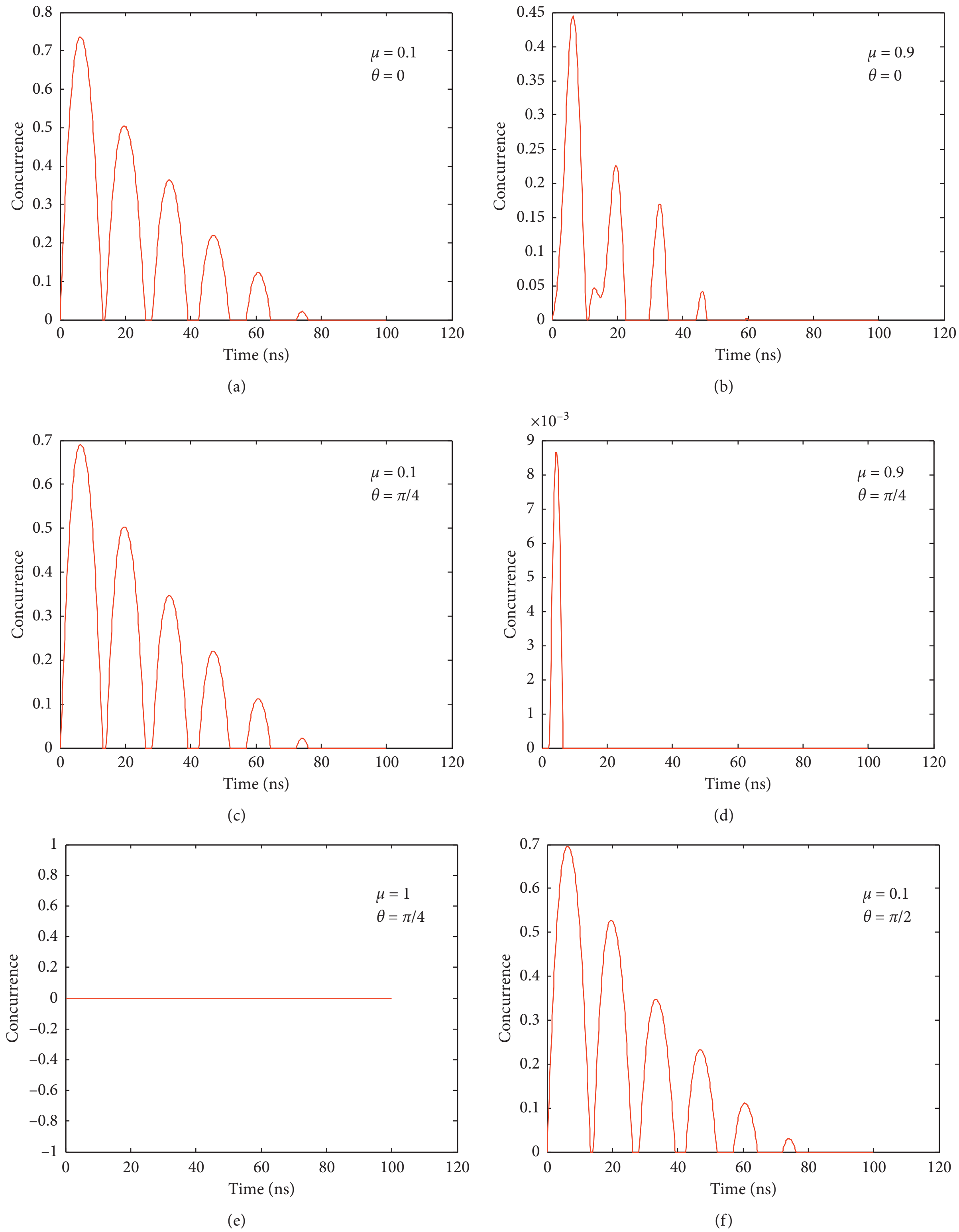
$$C(\rho) = \min \sum_i p_i C\left(\left|\psi_i\right\rangle\right). \tag{7}$$

A famous math formula was proposed by W. K. Wootters in 1998 for the concurrence of mixed states [29]:

$$C(\rho) = \max\left(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\right), \tag{8}$$

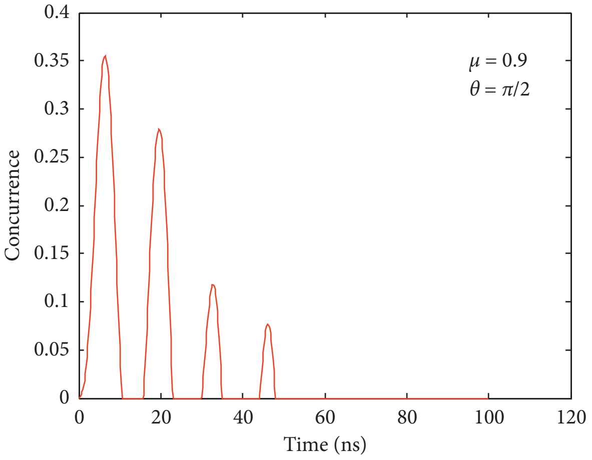
where λ_i $(i = 1, 2, 3, 4)$ denotes the square root of the eigenvalue of $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ in descending order, in which the operator σ_y is the Pauli operator and ρ^* is the complex conjugate of density matrix ρ of the coupled system. The simulation result is shown in Figure 3(b); the concurrence oscillates damply with microwave time. It is noteworthy that the concurrence exhibits death and revival, suggesting that the concurrence of the coupled system decreases or even disappears at certain microwave time when the system is experiencing decoherence. According to the simulation results, the concurrence of the coupled superconducting qubit and the TLS oscillates damply with microwave time and exhibits concurrence death and revival. Figure 3(b) shows that when the microwave driving time is about 60 ns, the concurrence will be completely lost and will not revival again, which reveals that the coupled system has been drowned in the ambient noise. Figure 3(b) suggests that the maximum concurrence is nearly 0.43 when the coupled system is under decoherence.

TLSs are usually considered a decoherence source that negatively affects the superconducting devices. However, they have longer coherence time than superconducting qubits, and they can also entangle superconducting qubits or other quantum



(a) (b) (c) (d) (e) (f)

FIGURE 4: Continued.



(g)

Figure 4: Time evolution of the concurrence for different purity of the initial states.

systems. According to the simulation results, the coupling effect between the qubit and TLS and the resonant microwave driving help to achieve concurrence. TLS and its coupling with superconducting phase qubit are the keys to the concurrence of the system. With more microwave driving time, the concurrence between the superconducting qubit and TLS decreases more rapidly or disappear completely (concurrence death). Thus, concurrence death and revival are dependent on the resonant microwave driving time that acts on the bipartite qubit-TLS system. Studying the concurrence of the bipartite superconducting qubit and TLS under the action of microwave helps to control the superconducting qubit in experiments.

It is confirmed that the coherence time of quantum system depends on the parameters of the circuit components, the coupling strength, the temperature, and purity of the initial states, etc. By tuning reasonable parameters, we can appropriately improve the coherence of superconducting quantum systems during experimental measurements. The entanglement of the coupled quantum system is the symbol of quantum correlation, which means the superconducting qubit has some correlation with the TLS when the system is under microwave field driving. In order to find out the entanglement mechanism of the superconducting quantum system, we confirm that the preparation of the initial state of the system has an important influence on the entanglement of the superconducting quantum system. The simulation results have shown that the entanglement of the coupled qubit and TLS system also depends on the purity of the initial states of the system. When the system is prepared in an entangled state, the entanglement differs when the system is initially set in a separated state. We assume the initial states of the coupled qubit and TLS system are extended Werner-like states, which is defined as follows [30]:

$$\rho(0) = \mu|\phi\rangle\langle\phi| + \frac{1}{4}(1-\mu)I,$$
$$|\phi\rangle = \cos\theta|0g\rangle + \sin\theta|1e\rangle, \tag{9}$$

where μ indicates the purity of the initial states of the qubit-TLS system, $0 \le \mu \le 1$, and I is the 4×4 unit matrix.

The concurrence of the coupled superconducting qubit and TLS is plotted for different initial states in Figure 4, and it is concluded that for a fixed value of θ, the larger the value of μ is, the faster the degree of entanglement decoherence will be. For $\mu = 1$ and $\theta = 0$, the initial state is in a separated state $|0g\rangle$, and the concurrence of the coupled system has been simulated in Figure 3(b). Furthermore, the sudden death and revival of entanglement occur for different initial states, and the time of entanglement sudden death occurs earlier and lasts longer for lager value of μ, which means the entanglement oscillates rapidly and decoherent until the entanglement leads to sudden death or dies completely [31, 32]. It must be emphasized that for $\mu = 1$, $\theta = \pi/4$, the initial states become the Bell states $\rho(0) = (\sqrt{2}/2)(|0g\rangle + |1e\rangle)$, and the concurrence of the coupled system is always 0 and does not exhibit concurrence sudden death and revival, which is shown in Figure 4(e).

4. Conclusion

In the present study, the quantum dynamics and entanglement mechanism of coupled superconducting qubit and TLS were demonstrated. According to the results, the coupling effect between the superconducting qubit and the TLS and the resonant microwave driving help to achieve concurrence, while concurrence death and revival are dependent on the resonant microwave driving time that acts on the coupled system. TLSs are always considered to negatively affect the manipulation of the superconducting qubit, whereas they exhibit longer coherent time than qubits. Thus, TLSs have a promising application of quantum memory. Furthermore, the simulation results show the entanglement of the coupled qubit and TLS system also depends on the purity of the initial states of the system. When the system is prepared in an entangled state, the entanglement differs when the system is initially set in a separated state, and entanglement sudden death and revival for different purity of the initial states have been simulated. The mentioned results are vital to control and measure the entanglement of a bipartite or multipartite quantum system in experiments.

Data Availability

The data used to support the findings of the study are included within the article and also available from the corresponding author upon request.

Conflicts of Interest

The author declares no conflicts of interest.

Acknowledgments

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