Asian Journal of Probability and Statistics

Volume 21, Issue 2, Page 22-34, 2023; Article no.AJPAS.96146
ISSN: 2582-0230

A New Inverse Pareto Distribution

Yassmen, Y. Abdelall [a*]

[a] *Department of Mathematical Statistics, Faculty of Graduate Studies for Statistical Research, Cairo University, Egypt.*

Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/AJPAS/2023/v21i2460

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/96146

Original Research Article

Received: 28/11/2022
Accepted: 30/01/2023
Published: 08/02/2023

Abstract

New distribution of inverse Pareto using kumaraswamy generalized distribution called Kumaraswamy generalized inverse Pareto distribution is introduced in this paper. The proposed model is applicable in the study of variety of fields. Several properties of the proposed distribution, including explicit expressions for the quantile, moments, moment generating function and are studied. The maximum likelihood estimation is used to estimate parameters model and to derive the information matrix of it. Simulation study is carried out to evaluate the estimated parameters. The performance of the new model is examined using real data set comparing with widely known distributions.

Keywords: *Kumaraswamy generalized; inverse Pareto distribution; quantile; moment generating function; maximum likelihood.*

1 Introduction

The Pareto distribution is the most popular model for analyzing heavy tailed phenomena. The Pareto distribution was first proposed by Pareto [1] as a model for the distribution of incomes and other financial variables, and other phenomena. The Pareto distribution has a wide range of applications in several fields such as life testing,

**Corresponding author: Email: yassmenamalek@gmail.com;*

economics, finance, engineering, and etc…[2]. Many generalized distributions of the Pareto distribution can be found in the literature. A few examples of these distributions are the beta-Pareto distribution by Akinsete et al. [3], the Kumaraswamy- Pareto distribution by Bourguignon et al. [4], the Kumaraswamy-generalized exponentiated Pareto distribution by Shams [5], the Kumaraswamy transmuted Pareto distribution by Chhetri et al. [6].

Klugman et al. [7] introduced the cumulative distribution function (cdf) of the two-parameter inverse Pareto (IP) distribution as follows:

$$G(x)=\left(\frac{x}{\beta+x}\right)^{\alpha}, \quad \alpha,\beta,x>0, \tag{1}$$

where α is shape parameter and β is scale parameter. Some authors deal with many families of some well-known distributions which are more flexible for modeling several types of data. Traditional models do not provide adequate fits to the real data with highly skewed. To solve this problem several, introducing several methods including additional one shape parameters, two shape parameters to generating new families of distributions are available in the statistical literature. Some well-known generators are: the generalized-exponential by Gupta and Kundo [8], Kumaraswamy generalized distribution by Cordeiro and de Castro [9], generalized beta-generated by Alexander et al. [10], weibull-generated by Bourguignon et al. [11], Kumaraswamy weibull-generated by Hassan and Elgarhy [12], generalized additive weibull-generated by Hassan et al. [13], inverse weibull-generated Hassan and Nassr [14], odd inverse Pareto-generated by Aldahlan et al. [15], modified weibull-generated by Abdelall [16] and more.

The goal of this article is to provide a new and more flexible distribution, called Kumaraswamy generalized inverse Pareto (KGIP) distribution and introduces expansions expressions for its cumulative and density functions. Numerous statistical properties of the KGIP distribution including quantile function, order statistics, moments and moment generating function are studied in second section. The third section provides parameter estimation using maximum likelihood method. The simulation study is introduced in fourth section. Fifth section displays the effectiveness of the proposed distribution by practical application on real data sets. Finally, the conclusion is provided.

2 Kumaraswamy Generalized Inverse Pareto Distribution

Cordeiro and de Castro [9] defined the cdf of the Kumaraswamy generalized distribution as follows:

$$F(x)=1-\left[1-\left(G(x)\right)^{a}\right]^{b}, \; a,b>0, \tag{2}$$

where a and b are additional shape parameters. Using (1), (2), the cdf of the Kumaraswamy generalized inverse Pareto (KGIP) distribution with parameter $\underline{\xi}=(\mathrm{a},\mathrm{b},\alpha,\beta)$ is given by:

$$F_{KGIP}(x,\underline{\xi})=1-\left[1-\left(\frac{x}{\beta+x}\right)^{\alpha a}\right]^{b}, \; x>0. \tag{3}$$

The corresponding probability density function (pdf), will be

$$f_{KGIP}(x,\underline{\xi})=a\,b\,\alpha\,\beta\,\frac{x^{\alpha a-1}}{(\beta+x)^{\alpha a+1}}\left[1-\left(\frac{x}{\beta+x}\right)^{\alpha a}\right]^{b-1}, \; \alpha,\beta,a,b>0, \; x>0, \tag{4}$$

where α, a and b are shape parameters and β is scale parameter.

2.1 Special distributions

Sub-models can be deduced from KGIP distribution as follows:
If $b = 1$ in (4), we get the exponentiated inverse Pareto distribution with parameters a, α, and β.
If $a = 1$ in (4), we get the exponentiated inverse Pareto distribution with parameters b, α, and β.
If $b = a = 1$ in (4), we get the inverse Pareto distribution with parameters α and β.

The survival and hazard rate functions of the KGIP distribution are obtained, respectively as follows:

$$S_{KGIP}(x, \underline{\xi}) = \left[1 - \left(\frac{x}{\beta + x}\right)^{\alpha a}\right]^{b},$$

And

$$h_{KGIP}(x, \underline{\xi}) = a\, b\, \alpha\, \beta \frac{x^{\alpha a - 1}}{(\beta + x)^{\alpha a + 1}} \left[1 - \left(\frac{x}{\beta + x}\right)^{\alpha a}\right]^{-1}.$$

Figs. 1 and 2 display the graphs of pdf and hazard function of the KGIP distribution.

2.2 Model expansions

Here, we give explicit expansions for the cdf and pdf of the KGIP model. By using the generalized binomial theorem

$$(1 + z)^{\nu} = \sum_{i=0}^{\infty} \binom{\nu}{i} z^{i}\ ,\ 0 < z < 1. \tag{5}$$

Equation (3) can be rewritten as follows:

$$F_{KGIP}(x) = 1 - \sum_{i=0}^{\infty} (-1)^{i} \binom{b}{i} \left(\frac{x}{\beta + x}\right)^{\alpha a i} = 1 - \sum_{i=0}^{\infty} \eta_i\ \tau(x, \underline{\theta})\ , \tag{6}$$

Where

$\eta_i = (-1)^{i} \binom{b}{i}$, and $\tau(x, \underline{\theta})$ denotes the Inverse Pareto cdf with parameters $\underline{\theta} = (\alpha a i, \beta)$. Now, using the power series (5) in the last term of (4), we get

$$\begin{aligned} f_{KGIP}(x) &= b \sum_{i=0}^{\infty} \frac{(-1)^{i}}{(i+1)} \binom{b-1}{i} \left(\frac{\alpha\, a (i+1) \beta\, x^{\alpha a (i+1) - 1}}{(\beta + x)^{\alpha a (i+1) + 1}}\right) \\ &= \sum_{i=0}^{\infty} k_i\ \psi(x, \underline{\vartheta}), \end{aligned} \tag{7}$$

where

$k_i = (-1)^i \binom{b}{i+1}$, and $\psi(x, \underline{\vartheta})$ is the Inverse Pareto density function with parameters $\underline{\vartheta} = (\alpha a(i+1), \beta)$.
Thus, the KGIP density function can be expressed as an infinite linear combination of the Inverse Pareto density. Thus, some of its statistical properties can be obtained directly from those properties of the Inverse Pareto distribution.

2.3 Quantile function and simulation

The quantile function of the KGIP distribution can be defined as follows:

$$Q(q) = \beta \left[1-(1-q)^{\frac{1}{b}}\right]^{\frac{1}{\alpha a}} \left[1-\left[1-(1-q)^{\frac{1}{b}}\right]^{\frac{1}{\alpha a}}\right]^{-1}, \quad 0<q<1. \tag{8}$$

Median of KGIP distribution can be obtained by putting q=0.5, that is

$$Q(0.5) = \beta \left[1-0.5^{\frac{1}{b}}\right]^{\frac{1}{\alpha a}} \left[1-\left[1-0.5^{\frac{1}{b}}\right]^{\frac{1}{\alpha a}}\right]^{-1}.$$

For simulating the KGIP random variable, let q be a uniform variate on the unit interval (0,1). Thus, by means of the inverse transformation method, we consider the random variable X given by:

$$X = \beta \left[1-(1-U)^{\frac{1}{b}}\right]^{\frac{1}{\alpha a}} \left[1-\left[1-(1-U)^{\frac{1}{b}}\right]^{\frac{1}{\alpha a}}\right]^{-1},$$

which follows (4), i.e, X distributed KGIP $(\mathrm{a}, \mathrm{b}, \alpha, \beta)$.

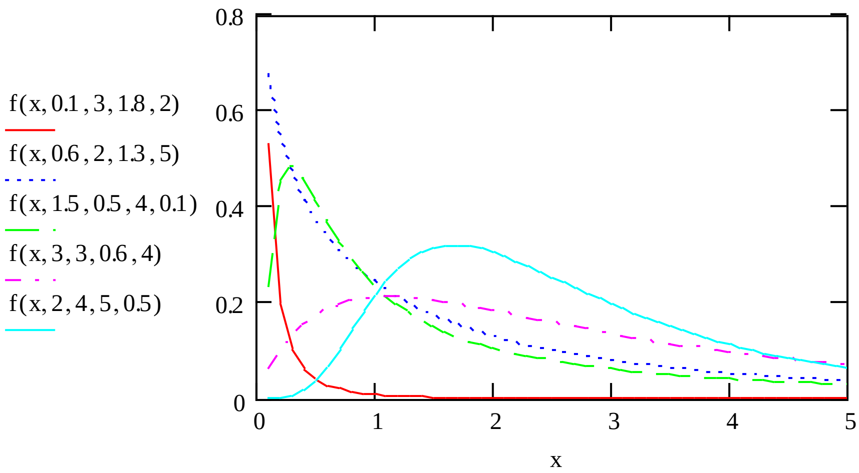

Fig. 1. The KGIP $(\mathrm{a}, \mathrm{b}, \alpha, \beta)$ density function

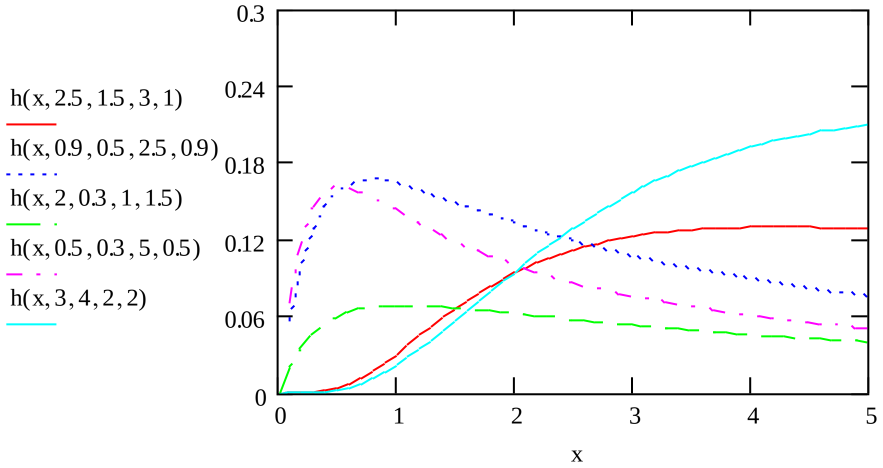

Fig. 2. The KGIP (a, b, α, β) hazard function

2.4 Skewness and Kurtosis

The Bowley Skewness measure introduced by Kenny and Keeping (1962) based on quantile function as follows:

$$BS = \frac{Q(3/4) - 2Q(1/2) + Q(1/4)}{Q(3/4) - Q(1/4)}.$$

Also, the Moors kurtosis measure introduced by Moors (1988) as follows:

$$MK = \frac{Q(7/8) - Q(5/8) + Q(3/8) - Q(1/8)}{Q(6/8) - Q(2/8)},$$

where $Q(.)$ is the quantile function. Fig. 3 illustrates plots of the skewness and kurtosis measures of the KGIP distribution for different values of the parameter b as a function of α and fixed values of a and β. These plots indicate that these measures decreases as $b = 0.5, 0.8, 1, 2$ (increases) for fixed a and β.

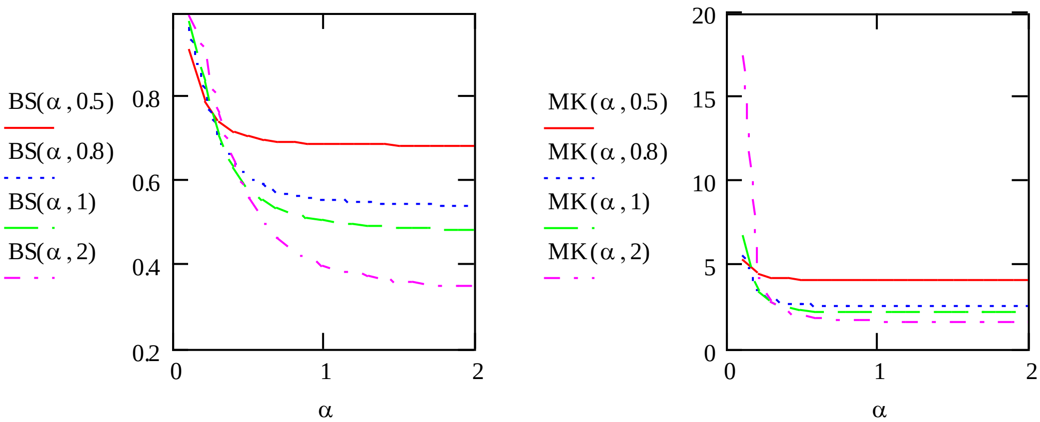

Fig. 3. KGIP skewness and kurtosis measures

2.5 Moments

The rthmoment for KGIP random variable X is given by:

$$\mu_r^{'} = E(X^r) = \int_{-\infty}^{\infty} x^r \; f_{KGIP}(x)\, dx$$

Using (7), we have

$$\mu_r^{'} = \sum_{i=0}^{\infty} k_i \int_0^{\infty} x^r \; \frac{\alpha\, a\,(i+1)\,\beta\, x^{\alpha\, a\,(i+1)-1}}{(\beta + x)^{\alpha\, a\,(i+1)+1}}\, dx,$$

Let $z = \dfrac{x}{(\beta + x)}$, after simplification, we obtain

$$\mu_r^{'} = \sum_{i=0}^{\infty} \alpha\, a\, \beta^r\, (i+1)\, k_i \int_0^1 z^{\alpha\, a\,(i+1)+r-1} (1-z)^{-r}\, dz,$$

Using beta function, we get

$$\begin{aligned}\mu_r^{'} &= \sum_{i=0}^{\infty} \alpha\, a\, \beta^r\, (i+1)\, k_i\; B(\alpha\, a\,(i+1) + r, 1-r) \\ &= \sum_{i=0}^{\infty} \beta^r k_i\, \frac{\Gamma(\alpha\, a\,(i+1)+r)\Gamma(1-r)}{\Gamma(\alpha\, a\,(i+1))}, \qquad -\alpha\, a\,(i+1) < r < 1,\end{aligned}$$

where $B(c,d) = \int_0^1 z^{c-1}(1-z)^{d-1} dz = \dfrac{\Gamma(c)\Gamma(d)}{\Gamma(c+d)}$ is the beta function, and $\Gamma(.)$ is the gamma function. If r is a negative integer, the rth moment is obtained as follows:

$$\mu_r^{'} = \sum_{i=0}^{\infty} \beta^r k_i\, \frac{(-r)!}{(\alpha\, a\,(i+1)-1)\ldots(\alpha\, a\,(i+1)+r)}.$$

The rth incomplete moment for KGIP random variable X is then equal to:

$$\begin{aligned} w_s(t) &= \int_0^t x^s \; f_{KGIP}(x)\, dx \\ &= \sum_{i=0}^{\infty} \alpha\, a\, \beta^s\, (i+1)\, k_i\; B\left(\frac{t}{\beta + t},\; \alpha\, a\,(i+1) + s,\; 1-s\right), \quad -\alpha\, a\,(i+1) < r < 1, \end{aligned} \tag{9}$$

where $B(x,c,d) = \int_0^x z^{c-1}(1-z)^{d-1} dz$ is the incomplete beta function. Also, the moment generating function of the proposed distribution is defined as:

$$M_X(t) = E\left[e^{tx}\right] = \sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r^{'} = \sum_{r,i=0}^{\infty} k_i \frac{(t\beta)^r \Gamma(\alpha a (i+1) + r)\Gamma(1-r)}{\Gamma(\alpha a (i+1))\Gamma(r+1)}.$$

2.6 Order statistic

Taking a simple random sample of size n from KGIP($\underline{\xi}$) with cdf and pdf $F_{KGIP}(x;\underline{\xi})$ and $f_{KGIP}(x;\underline{\xi})$ given by (3) and (4) respectively. The ordered statistics sample obtained from random sample is $X_{1:n} \le X_{2:n} \le ..., X_{n:n}$. The pdf of the rth order statistic $X_{r:n}$ is:

$$f_{r:n}(x;\underline{\xi}) = \frac{1}{B(r, n-r+1)} f(x;\underline{\xi})\left[F(x;\underline{\xi})\right]^{r-1}\left[1-F(x;\underline{\xi})\right]^{n-r}, \tag{10}$$

where B(.,.) is the beta function. Since $0 < F(x;\underline{\xi}) < 1$ for $x > 0,$ we can use the binomial expansion of $\left[1-F(x;\underline{\xi})\right]^{n-r}$ given as follows

$$\left[1-F(x;\underline{\xi})\right]^{n-r} = \sum_{i=0}^{n-r}\binom{n-r}{i}(-1)^i\left[F(x;\underline{\xi})\right]^i. \tag{11}$$

Substituting from (11) into (10), we obtain

$$f_{r:n}(x;\underline{\xi}) = \frac{1}{B(r, n-r+1)} f(x;\underline{\xi})\sum_{i=0}^{n-r}\binom{n-r}{i}(-1)^i\left[F(x;\underline{\xi})\right]^{i+r-1}. \tag{12}$$

Based on (6) and (7), we can write

$$f(x,\underline{\xi})\,F(x,\underline{\xi})^{i+r-1} = a\,b\sum_{k,j,t,l=0}^{\infty}\frac{(-1)^{k+l}\alpha\,\beta}{(\beta+x)^{\alpha\,a\,(l+1)+1}}\binom{(i+r-1)}{k}\binom{b(l+1)-1}{l}x^{\alpha\,a(l+1)-1}. \tag{13}$$

Inserting (13) in (12), the pdf of $X_{r:n}$ written as

$$f_{r;n}(x,\underline{\xi}) = \sum_{k,l,i=0}^{\infty}\omega_{k,l,i}\,\psi(x,\underline{\vartheta}),$$

where $\omega_{k,j,t,l} = \sum_{i=0}^{n-r}\frac{(-1)^{k+l+i}a\,b}{B(r,n-r+1)}\binom{n-r}{i}\binom{(i+r-1)}{k}\binom{b(l+1)-1}{l}$, $\psi(x,\underline{\vartheta})$ denotes the Inverse Pareto distribution with parameters $\underline{\vartheta} = (\alpha a(l+1), \beta)$. Thus, the pdf of the KGIP order statistics is a linear combination of the Inverse Pareto density. Also we can define first order statistics $X_{1:n} = \min(X_1; X_2;...; X_n)$, and the last order statistics as $X_{n:n} = \max(X_1; X_2;...; X_n)$.

2.7 Mean deviations

The useful measures of variation for population are mean deviations about mean and median. If the KGIP has mean (μ) and median (m), the mean deviations about mean and median are respectively, can be defined as:

$$D(\mu)=2\mu\, F_{KGIP}(\mu)-2\, w_{1\,KGIP}(\mu), \text{ and } D(m)=\mu-2\, w_{1\,KGIP}(m),$$

where $w_1(.)$ is the first incomplete moment of KGIP obtained from (9) for $s=1$.

3 Parameter Estimation of KGIP Distribution

Using the maximum likelihood estimation, we derived the estimated parameters of the KGIP distribution. Let $X_1, X_2,..., X_n$ be a random sample from KGIP$(\underline{\xi})$ where $\underline{\xi}=(a, b, \alpha, \beta)$ be the vector of the parameters, the log-likelihood function, ℓ, is given by:

$$\ell(\underline{\xi})= n\log a + n\log b + n\log\alpha + n\log\beta + (\alpha\, a-1)\sum_{i=1}^{n}\log(x_i)-(\alpha\, a+1)\sum_{i=1}^{n}\log(\beta+x_i)$$
$$+(b-1)\sum_{i=1}^{n}\log\left[1-\left(\frac{x_i}{\beta+x_i}\right)^{\alpha\, a}\right]. \quad (14)$$

The score vector $U(\underline{\xi})=(\partial\ell/\partial a, \partial\ell/\partial b, \partial\ell/\partial\alpha, \partial\ell/\partial\beta)^T$, where the components to the parameters in $\underline{\xi}$ are given by differentiating (14). By setting $z_i = x_i\,(\beta+x_i)^{-1}$,

$$U_a=\frac{n}{a}+\alpha\sum_{i=1}^{n}\log(x_i)-\alpha\sum_{i=1}^{n}\log(\beta+x_i)-(b-1)\sum_{i=1}^{n}\left[\frac{\alpha\, z_i^{\alpha\, a}\log(z_i)}{(1-z_i^{\alpha\, a})}\right], \quad (15)$$

$$U_b=\frac{n}{b}+\sum_{i=1}^{n}\log(1-z_i^{\alpha\, a}), \quad (16)$$

$$U_\alpha=\frac{n}{\alpha}+a\sum_{i=1}^{n}\log(x_i)-a\sum_{i=1}^{n}\log(\beta+x_i)-(b-1)\sum_{i=1}^{n}\left[\frac{a\, z_i^{\alpha\, a}\log(z_i)}{(1-z_i^{\alpha\, a})}\right], \quad (17)$$

And

$$U_\beta=\frac{n}{\beta}-(\alpha\, a+1)\sum_{i=1}^{n}(\beta+x_i)^{-1}+(b-1)\sum_{i=1}^{n}\left[\frac{\alpha\, a\, z_i^{\alpha\, a}}{(\beta+x_i)(1-z_i^{\alpha\, a})}\right]. \quad (18)$$

The maximum likelihood estimates (MLEs) of the unknown parameters are obtained by setting system of non-linear (15) - (18) equations to zero and solve them simultaneously. From equation (16), The MLE of b can be rewritten in the closed form as follows

$$\hat{b}=\frac{-n}{\sum_{i=1}^{n}\log\left[1-z_i^{\hat{\alpha}\,\hat{a}}\right]}, \quad (19)$$

When estimates $(\hat{a},\hat{\alpha})$ known. Substituting from (19) into (15), (17), and (18), we get the MLEs $(\hat{a},\hat{\alpha},\hat{\beta})$. These equations cannot be solved analytically and statistical software can be used to solve them simultaneously. For interval estimation of the parameters, we require the 4×4 observed information matrix $J(\underline{\xi})=\{U_{s,t}\}$, where $s,t=(a,b,\alpha,\beta)$ given in Appendix A. To construct the approximate confidence intervals for the parameters a,b,α, and β of the KGIP distribution, the multivariate normal $N_4\left(0, J(\hat{\underline{\xi}})^{-1}\right)$ distribution can be used. Here, $J(\hat{\underline{\xi}})$ is the total observed information matrix evaluated at $\hat{\underline{\xi}}$. The asymptotic $100(1-\eta)\%$ confidence intervals for parameters a,b,α, and β are respectively given by:

$$\hat{a} \pm Z_{\eta/2}\sqrt{\operatorname{var}(\hat{a})},\ \hat{b} \pm Z_{\eta/2}\sqrt{\operatorname{var}(\hat{b})},\ \hat{\alpha} \pm Z_{\eta/2}\sqrt{\operatorname{var}(\hat{\alpha})}, \text{and } \hat{\beta} \pm Z_{\eta/2}\sqrt{\operatorname{var}(\hat{\beta})},$$

where $Z_{(\eta/2)}$ is the upper $(\eta/2)^{th}$ percentile of the standard normal distribution and the $Var(.)'s$ denote the diagonal elements of $J(\hat{\underline{\xi}})^{-1}$ corresponding to a, b, α, and β.

4 Simulation Study

Simulation study has been performed for average MLEs, Mean Square Error (MSE), and Bias. The KGIP random number generation was performed of size $n=50(50)200$ for $N=1000$ replications. The parameter values used in the data generation processes are followed in Table 1. Bias and MSE are calculated by:

$$Bias=\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\xi}_i-\xi\right),\quad MSE=\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\xi}_i-\xi\right)^2,$$

where $\xi=(a,b,\alpha,\beta)$. Simulation results were obtained for several combinations of parameters. Table 1 lists the means of the MLEs of the four parameters KGIP with respective MSEs, and Biases. It can be illustrated from table that, the MSEs and Biases decreases for all parameter combinations when sample size increase.

5 Data Analysis

The importance and flexibility of the KGIP distribution can be examined by application to real data set. We compare the fit of KGIP with some competitive distributions namely: the Kumaraswamy-Pareto (KP), Exponentiated Pareto (EP), Beta Pareto (BP), and Pareto distributions [4]. The data set correspond to the exceedances of flood peaks of the Wheaton River near Careross in Yukon Territory, Canada. The data consist of 72 excessdances for the years 1958-1984, rounded to one decimal place. They were analyzed by Choulakian and Stephens [17] and are listed in Table 2.

In order to compare the distributions, we calculated the MLEs of the parameters, -2 log-likelihood (-2ℓ) and the goodness of fit statistics measures namely: Akaike Information Criteria (AIC), and Bayesian Information Criteria(BIC), Consistent Akaike Information Criteria (CAIC), and Kolmogorov-Smirnov statistic (K-S) for all distributions and listed in Table 3. The KGIP distribution has the lowest AIC, CAIC, BIC and K-S among all the fitted other distributions. All required computations are performed using Mathcad package. From the results, the proposed distribution is the best model under these data than other competitive distributions.

Table 1. Mean estimates, corresponding MSE and Bias of the MLE of (a, b, α, β)

Actual Values	n	Mean				Bias				MSE			
		$(\hat{a})$	$(\hat{b})$	$(\hat{\alpha})$	$(\hat{\beta})$	$(\hat{a})$	$(\hat{b})$	$(\hat{\alpha})$	$(\hat{\beta})$	$(\hat{a})$	$(\hat{b})$	$(\hat{\alpha})$	$(\hat{\beta})$
$a = 0.5$	50	0.734	1.955	2.158	0.509	0.234	1.455	0.668	0.093	0.055	2.122	0.437	0.00014
$b = 0.5$	100	0.716	1.887	2.160	0.511	0.216	1.387	0.650	0.017	0.047	1.924	0.436	0.00013
$\alpha = 1.5$	150	0.714	1.861	2.140	0.510	0.213	1.361	0.640	0.011	0.046	1.853	0.410	0.00012
$\beta = 0.5$	200	0.707	1.848	2.130	0.509	0.207	1.348	0.631	0.010	0.043	1.818	0.395	0.00012
$a = 0.5$	50	1.249	1.955	1.250	0.509	0.749	0.455	0.750	0.093	0.561	0.212	0.562	0.00015
$b = 1.5$	100	1.236	1.885	1.237	0.511	0.736	0.387	0.737	0.018	0.542	0.150	0.543	0.00014
$\alpha = 0.5$	150	1.232	1.860	1.233	0.511	0.732	0.361	0.733	0.015	0.536	0.131	0.537	0.00013
$\beta = 0.5$	200	1.229	1.848	1.229	0.510	0.728	0.348	0.729	0.012	0.531	0.121	0.532	0.00012
$a = 0.5$	50	1.249	1.955	1.249	0.509	0.749	1.455	0.749	0.009	0.562	2.122	0.561	0.00015
$b = 0.5$	100	1.236	1.887	1.237	0.510	0.736	1.387	0.737	0.016	0.542	1.924	0.543	0.00014
$\alpha = 0.5$	150	1.232	1.861	1.233	0.510	0.732	1.361	0.733	0.014	0.537	1.853	0.536	0.00012
$\beta = 0.5$	200	1.229	1.848	1.229	0.509	0.729	1.348	0.729	0.011	0.531	1.818	0.532	0.00011
$a = 0.5$	50	0.721	1.955	2.163	0.509	0.221	0.455	0.663	0.093	0.049	0.212	0.441	0.00015
$b = 1.5$	100	0.714	1.887	2.142	0.511	0.214	0.387	0.642	0.018	0.046	0.150	0.412	0.00014
$\alpha = 1.5$	150	0.712	1.861	2.135	0.510	0.212	0.361	0.635	0.011	0.045	0.131	0.403	0.00013
$\beta = 0.5$	200	0.710	1.848	2.129	0.510	0.210	0.348	0.629	0.014	0.044	0.121	0.395	0.00012
$a = 0.5$	50	0.703	2.507	2.011	0.777	0.203	1.007	0.511	-0.023	0.041	1.023	0.262	0.00063
$b = 1.5$	100	0.693	2.424	2.002	0.776	0.193	0.924	0.508	-0.024	0.037	0.856	0.259	0.00059
$\alpha = 1.5$	150	0.687	2.395	2.008	0.776	0.187	0.895	0.505	-0.024	0.035	0.801	0.257	0.00052
$\beta = 0.8$	200	0.685	2.382	2.005	0.776	0.185	0.882	0.502	-0.025	0.034	0.778	0.255	0.00051
$a = 0.5$	50	0.608	3.991	1.874	1.873	0.114	2.684	0.386	-0.124	0.013	6.211	0.149	0.018
$b = 1.5$	100	0.614	4.184	1.882	1.876	0.108	2.491	0.384	-0.127	0.012	6.010	0.147	0.017
$\alpha = 1.5$	150	0.603	3.931	1.883	1.870	0.103	2.431	0.383	-0.131	0.011	5.912	0.146	0.016
$\beta = 2$	200	0.601	3.908	1.886	1.870	0.101	2.408	0.374	-0.133	0.010	5.800	0.144	0.014

Table 2. Exceedances of Wheaton River flood data

1.7	2.2	14.4	1.1	0.4	20.6	5.3	0.7	1.9	13.0	12.0	9.3
1.4	18.7	8.5	25.5	11.6	14.1	22.1	1.1	2.5	14.4	1.7	37.6
0.6	2.2	39.0	0.3	15.0	11.0	7.3	22.9	1.7	0.1	1.1	0.6
9.0	1.7	7.0	20.1	0.4	2.8	14.1	9.9	10.4	10.7	30.0	3.6
5.6	30.8	13.3	4.2	25.5	3.4	11.9	21.5	27.6	36.4	2.7	64.0
1.5	2.5	27.4	1.0	27.1	20.2	16.8	5.3	9.7	27.5	2.5	27.0

Table 3. MLEs,-2log-likelihood, AIC, CAIC, and K-S

Model	**a**	**b**	α	β	-2ℓ	**AIC**	**BIC**	**CAIC**	**K-S**
Pareto	1	1	0.2438	0.1	606.2	608.2	610.4	608.2	0.332
EP	2.8797	1	0.4241	0.1	574.6	578.6	583.2	578.8	0.198
BP	3.1473	85.7508	0.0088	0.1	567.4	573.4	580.3	573.8	0.174
KP	2.8553	85.8468	0.0528	0.1	542.4	548.4	555.3	548.8	0.170
KGIP	**1.112**	**2.712**	**0.865**	**2.191**	**507.7**	**515.8**	**524.9**	**516.4**	**0.168**

6 Conclusion

In this article, we introduce a new generalization of the inverse Pareto distribution, termed as KGIP distribution, which can be quite flexible in analyzing real data in different fields. Study several statistical properties of the proposed distribution. Maximum likelihood estimation has been used to estimate the model parameters. The simulation study is used to assess the performance of the estimated parameters. The applicability of the proposed distribution is examined by applying to a real data set. The KGIP distribution provides better fit than other compared distributions.

Competing Interests

Author has declared that no competing interests exist.

References

[1] Pareto V. Cours economic politique. Lausanne and Paris, Range and Cie; 1986.

[2] Evans M, Hastings N, Peacock P. Statistical distributions. Third edition, John Wiley& Sons, Inc. New York; 2000.

[3] Akinsete A, Famoye F, Lee C. The beta-Pareto distribution. Statistics. 2008;42(6):547-563.

[4] Bourguignon M, Silva RB, Zea LM, Cordeiro GM. The Kumaraswamy Pareto distribution. Journal of Statistical Theory and Applications. 2013;12(2).129-144.

[5] Shams TM. The Kumaraswamy-generalized exponentiated pareto distribution. European Journal of Applied Sciences. 2013;5(3):92-99.

[6] Chhetri SB, Akinsete AA, Aryal G, Long H. The Kumaraswamy transmuted Pareto distribution. Journal of Statistical Distributions and Applications. 2017;4(11):1-24.

[7] Klugman SA, Panjer HH, Willmot GE. Loss models: from data to decisions. Third edition, John Wiley& Sons, Hoboken. New Jersey.

[8] Gupta RA, Kundo D. Generalized exponential distribution. Statistics. 1999;41(2):173-188.

[9] Cordeiro GM, de Castro M. A new family of generalized distributions. Journal of Statistical Computation and Simulation. 2011;81:883-898.

[10] Alexander C, Cordeiro GM, Ortega EMM, Sarabia JM. Generalized beta-generated distributions. Computational Statistics & Data Analysis. 2012;56:1880-1897.

[11] Bourguignon M, Silva RB, Cordeiro GM. The Weibull-G family of probability distributions. Journal of Data Science. 2014;12:53-68.

[12] Hassan AS, Elgarhy M. Kumaraswamy Weibull-generated family of distributions with applications. Advances and Applications in Statistics. 2016;48:205-239.

[13] Hassan AS, Hemeda SE, Maiti SS, Pramanik S. The generalized additive Weibull-G family of probability distributions. International Journal of Statistics and Probability. 2017;6(5):65-83.

[14] Hassan AS, Nassr SG. The inverse Weibull generator of distributions: properties and applications. Journal of Data Science. 2018;723-742.

[15] Aldahlan MA, Afify AZ, Ahmed AN. The odd inverse Pareto-G: Properties and applications. Journal of Nonlinear Sciences and Applications. 2019;12:278-290.

[16] Abdelall YY. The modified weibull-G family of distributions: Properties and applications. International Journal of Mathematical Archive. 2019;10(4):21-31.

[17] Choulakian V, Stephens MA. Goodness-of-fit for the generalized Pareto distribution. Technometrics. 2001;43:478-484.

Appendix A

Elements of the information matrix $J(\underline{\xi}) = \{U_{s,t}\}$, where $s,t = (\mathrm{a}, \mathrm{b}, \alpha, \beta)$ are given by:

$$U_{aa} = \frac{-n}{a^2} - (b-1)\sum_{i=1}^{n}\left[\frac{\alpha^2 z_i^{\alpha a}\log(z_i)^2}{(1-z_i^{\alpha a})}\left[1+\frac{z_i^{\alpha a}}{(1-z_i^{\alpha a})}\right]\right],$$

$$U_{ab} = -\alpha\sum_{i=1}^{n}\left[\frac{z_i^{\alpha a}\log(z_i)}{(1-z_i^{\alpha a})}\right],$$

$$U_{a\alpha} = \sum_{i=1}^{n}\log(x_i) - \sum_{i=1}^{n}\log(\beta+x_i) - (b-1)\sum_{i=1}^{n}\left[\frac{z_i^{\alpha a}\log(z_i)}{(1-z_i^{\alpha a})}\left[1+\alpha\, a\log(z_i)+\frac{\alpha\, a\, z_i^{\alpha a}\log(z_i)}{(1-z_i^{\alpha a})}\right]\right],$$

$$U_{a\beta} = -\alpha\sum_{i=1}^{n}(\beta+x_i)^{-1} + (b-1)\sum_{i=1}^{n}\left[\frac{\alpha\, z_i^{\alpha a}}{(\beta+x_i)(1-z_i^{\alpha a})}\left[1+a\log(z_i)+\frac{\alpha\, a\, z_i^{\alpha a}\log(z_i)}{(1-z_i^{\alpha a})}\right]\right],$$

$$U_{bb} = \frac{-n}{b^2},$$

$$U_{b\alpha} = -a\sum_{i=1}^{n}\left[\frac{z_i^{\alpha a}\log(z_i)}{(1-z_i^{\alpha a})}\right],$$

$$U_{b\beta} = \alpha\, a\sum_{i=1}^{n}\left[\frac{z_i^{\alpha a}}{(\beta+x_i)(1-z_i^{\alpha a})}\right],$$

$$U_{\alpha\alpha} = \frac{-n}{\alpha^2} - (b-1)\sum_{i=1}^{n}\left[\frac{a^2 z_i^{\alpha a}\log(z_i)^2}{(1-z_i^{\alpha a})}\left[1+\frac{z_i^{\alpha a}}{(1-z_i^{\alpha a})}\right]\right],$$

$$U_{\alpha\beta} = -a\sum_{i=1}^{n}(\beta+x_i)^{-1} + (b-1)\sum_{i=1}^{n}\left[\frac{a\, z_i^{\alpha a}}{(\beta+x_i)(1-z_i^{\alpha a})}\left[1+\alpha\, a\log(z_i)+\frac{\alpha\, a\, z_i^{\alpha a}\log(z_i)}{(1-z_i^{\alpha a})}\right]\right], \text{ and}$$

$$U_{\beta\beta} = \frac{-n}{\beta^2} + (\alpha\, a+1)\sum_{i=1}^{n}(\beta+x_i)^{-2} - (b-1)\sum_{i=1}^{n}\left[\frac{\alpha\, a\, z_i^{\alpha a}}{(\beta+x_i)^2(1-z_i^{\alpha a})}\left[(\alpha\, a+1)+\frac{z_i^{\alpha a}}{(1-z_i^{\alpha a})}\right]\right].$$

where $z_i = x_i(\beta+x_i)^{-1}$.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
https://www.sdiarticle5.com/review-history/96146