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Transmuted Ailamujia Distribution with Applications to Lifetime Observations

A. A. Adetunji [a*]

[a] *Department of Statistics, Federal Polytechnic, Ile-Oluji, Nigeria.*

Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

Background of the Study: Quest to extend existing probability distributions to allow for more flexibility in data modelling is never ending.
Aims: This study extends the Ailamujia distribution using quadratic transmutation map to propose a new 2-paramter lifetime distribution.
Methodology: Shapes of the density function and the hazard rate function of the new propositions are obtained. Some mathematical properties of the new distribution are also obtained.
Results: Application of the distribution to four lifetime datasets reveals that the distribution competes favourably with other related distributions.
Conclusion: The new distribution competes favourably with the existing distributions in data modelling.

Keywords: Ailamujia distribution; quadratic transmutation; lifetime observations; reliability functions; parameter estimation.

**Corresponding author: Email: adecap4u@gmail.com;*

1 Introduction

In attempts to better explain various lifetime phenomena, different extensions to several baseline continuous lifetime distributions have been proposed. In most cases, these propositions involve extending/compounding baseline distributions by adding extra parameter(s) to improve flexibilities and general applicability. Among these extension techniques are the Beta G [1]; Quadratic Transmutation [2]; Kumaraswamy G [3]; Exponentiated Generalized G [4]; and Alpha Power Transformation [5].

In recent times, the Ailamujia distribution [6] is receiving attentions among different baseline lifetime distributions due to its general applicability to unimodal lifetime observations with an increasing hazard rate [7]. The distribution corresponds to the Erlang distribution with shape parameter of 2 and rate parameter θ. In improving the flexibility of the distribution, different generalized forms have been proposed. The Weighted Ailamujia distribution was introduced by [8] while [9] proposed its Size-biased form. Also, [10] proposed the Marshall-Olkin extension; [7] introduced the Power Ailamujia distribution and [11] proposed the Odd-Weibull generalized version.

In this study, we propose a new two-parameter extension of the Ailamujia distribution using the quadratic transmutation map [2].

2 Methodology

2.1 Quadratic transmutation map

If the CDF of a baseline distribution is denoted with $G(x)$, the Quadratic Transmuted form of the distribution is defined as:

$$F(x) = (1+a)G(x) - a\big(G(x)\big)^2; \qquad x \in \mathbb{R}, |a| \leq 1 \tag{1}$$

The CDF in (1) has attracted various interest among researchers leading to the extension of several baseline lifetime distributions. [12] provided list of quadratic transmuted distributions while [13] provided detailed review on the quadratic transmutation map and its application to extend various lifetime distributions

2.2 The ailamujia distribution

This is a single parameter lifetime distribution [6] defined with the distribution function:

$$G(x) = 1 - (1+\theta x)e^{-\theta x} \;,\; x \geq 0 \tag{2}$$

The resulting density function of (2) is given as:

$$g(x) = \theta^2 x e^{-\theta x} \;,\; x \geq 0 \tag{3}$$

2.3 The quadratic transmuted ailamujia distribution

The Quadratic Transmuted Ailamujia Distribution (QTAD) is obtained by inserting (2) into (1). Hence, the CDF of the QTAD is obtained as:

$$F(x) = 1 - a(1+\theta x)^2 e^{-2\theta x} + (a-1)(1+\theta x)e^{-\theta x} \tag{4}$$

The corresponding probability distribution function is obtained as:

$$f(x) = \theta^2 x e^{-\theta x}\big(1 - a + 2ae^{-\theta x} + 2a\theta x e^{-\theta x}\big) \tag{5}$$

Special cases

1. If the transmutation parameter $a = 0$, the distribution in (5) becomes the Ailamujia Distribution [6].
2. The distribution becomes a new 1-parameter distribution when $a = 1$, with the new PDF obtained as $2\theta^2 x e^{-2\theta x}(1 + \theta x)$

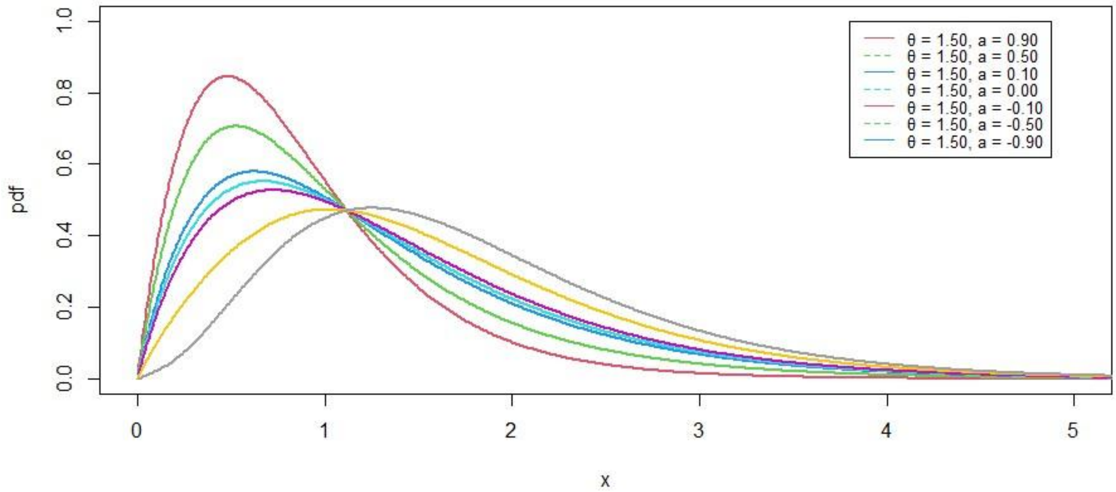

Fig. 1. Shapes of the PDF of QTAD for different values of *a* when θ = 1.50

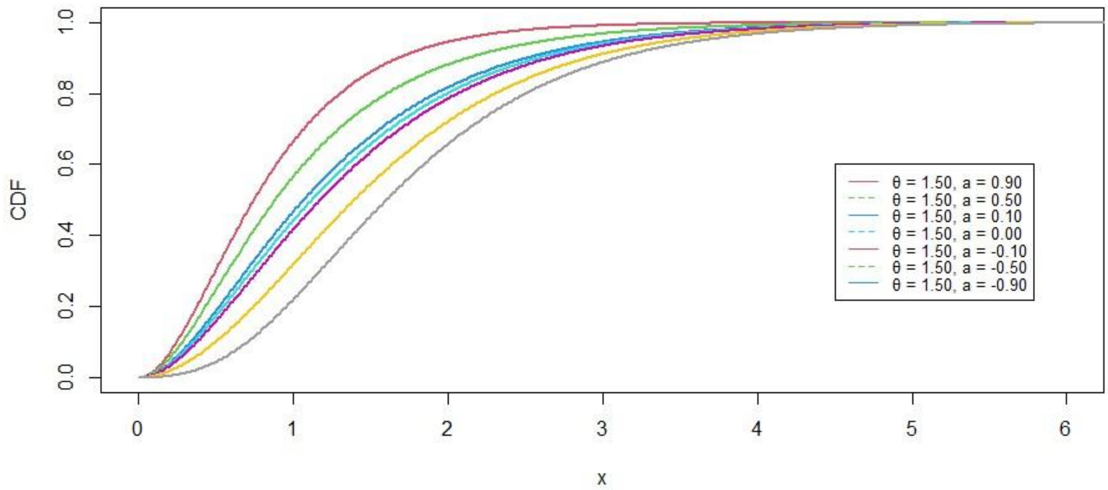

Fig. 2. Shapes of the CDF of QTAD for different values of *a* when θ = 1.50

Different possible shapes of PDF and CDF of the QTAD are respectively illustrated in Figs. 1 and 2. Fig. 1 reveals that the distribution is unimodal and positively skewed with possibility of effectively modelling observations with varying skewness and kurtosis. The curve flattens towards normality as the transmutation parameter $a \to -1$.

2.4 Reliability functions

While the survival function $S(x) = 1 - F(x)$ reveals the probability of a phenomenon not failing preceding a particular time, the hazard rate function *h(x)* is the risk of a system experiencing a particular event in an instantaneous time. It is defined as: $h(x) = \frac{f(x)}{S(x)}$.

The survival and hazard rate functions of the QTAD are obtained as:

$$S(x) = a(1+\theta x)^2 e^{-2\theta x} - (a-1)(1+\theta x)e^{-\theta x} \tag{6}$$

$$h(x) = \frac{\theta^2 x\left(2a\theta x e^{-\theta x} + 2ae^{-\theta x} - a + 1\right)}{\left(a(1+\theta x)e^{-\theta x} - a + 1\right)(1+\theta x)} \tag{7}$$

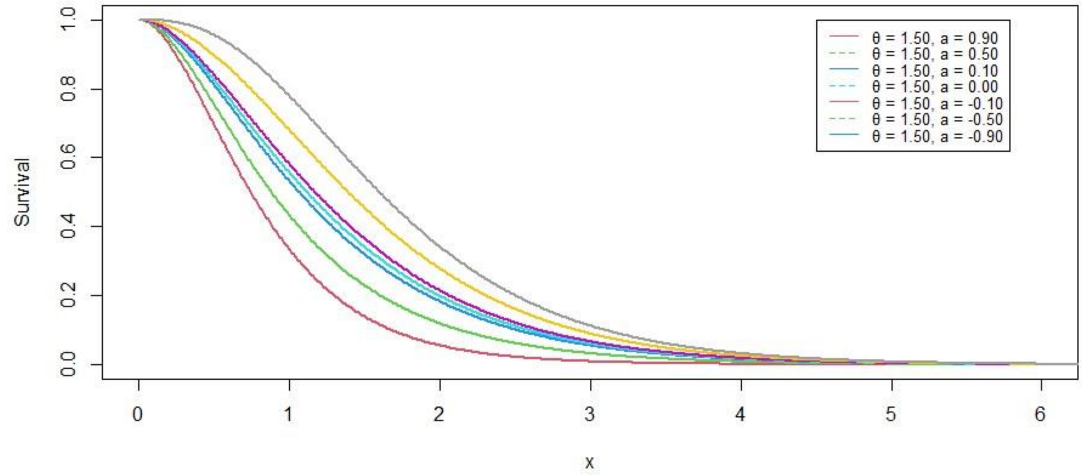

Fig. 3. Shapes of the Survival function of QTAD for different values of *a* when θ = 1.50

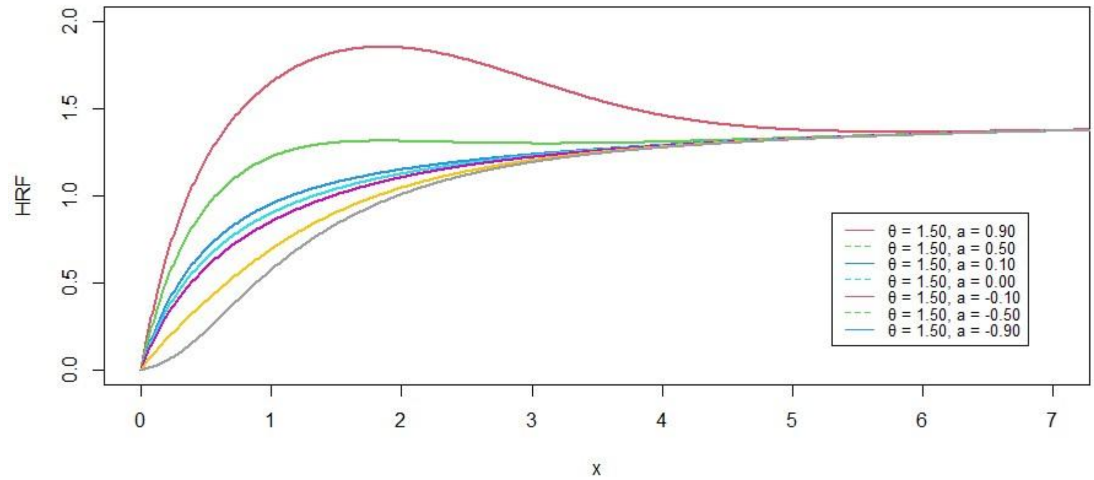

Fig. 4. Shapes of the hazard rate function of QTAD for different values of *a* when θ = 1.50

Figs. 3 and 4 respectively illustrate the survival and hazard rate function of the QTAD for varying transmutation parameter at constant θ.

3 Mathematical Properties

3.1 Order statistics

If a continuous random variable X has the QTAD as defined in (5) and if $X_{1,n} < X_{2,n} < \cdots < X_{n,n}$ is a set of ordered random variable of size n, then the distribution function of the k^{th} ordered statistics of X is defined as:

$$f_{k,n}(x) = \frac{n!}{(k-1)!(n-k)!} f(x)[F(x)]^{k-1}[S(x)]^{n-k}.$$

Hence, the k^{th} ordered statistics of $X \sim QTAD(\theta, a)$ is obtained as:

$$f_{k,n}(x) = \frac{n!}{(k-1)!(n-k)!}\left(\theta^2 x e^{-\theta x}\left(1 - a + 2ae^{-\theta x} + 2a\theta x e^{-\theta x}\right)\right)\left[1 - a(1+\theta x)^2 e^{-2\theta x} + \right.$$
$$a - 11 + \theta x e - \theta x\; k - 1a1 + \theta x 2e - 2\theta x - a - 11 + \theta x e - \theta x n - k \quad (8)$$

The 1^{st} and n^{th} ordered statistic for X are given in (9) and (10) for $k = 1$ and $k = n$ respectively.

$$f_{1,n}(x) = n\theta^2 x e^{-\theta x}\left(1 - a + 2ae^{-\theta x} + 2a\theta x e^{-\theta x}\right)\left[a(1+\theta x)^2 e^{-2\theta x} - (a-1)(1+\theta x)e^{-\theta x}\right]^{n-1} \quad (9)$$

$$f_{k,n}(x) = n\theta^2 x e^{-\theta x}\left(1 - a + 2ae^{-\theta x} + 2a\theta x e^{-\theta x}\right)\left[1 - a(1+\theta x)^2 e^{-2\theta x} + (a-1)(1+\theta x)e^{-\theta x}\right]^{n-1} \quad (10)$$

3.2 Quantiles function

If $X \sim QTAD(\theta, a)$, then the quantile function X is obtained given as:

$$Q_{(u)} = -\frac{1}{\theta}\boldsymbol{W}\left(\frac{e^{-1}\left(1 - a + \sqrt{a^2 - 4au + 2a + 1}\right)}{2a}\right) - \frac{1}{\theta} \quad (11)$$

The Lambert function $\boldsymbol{W}$ is a multiple-values complex function defined as $W_{(u)}e^{W(u)} = u$. Hence, the first three quartiles $\left(Q_{\left(\frac{1}{4}\right)}, Q_{\left(\frac{1}{2}\right)}, Q_{\left(\frac{3}{4}\right)}\right)$ of X are obtained by setting u to ¼, ½, and ¾ respectively in (11). These are obtained in (12), (13), and (14) respectively.

$$Q_{\left(\frac{1}{4}\right)} = -\frac{1}{\theta}\boldsymbol{W}\left(\frac{e^{-1}\left(1 - a + \sqrt{a^2 + a + 1}\right)}{2a}\right) - \frac{1}{\theta} \quad (12)$$

$$Q_{\left(\frac{1}{2}\right)} = -\frac{1}{\theta}\boldsymbol{W}\left(\frac{e^{-1}\left(1 - a + \sqrt{a^2 + 1}\right)}{2a}\right) - \frac{1}{\theta} \quad (13)$$

$$Q_{\left(\frac{3}{4}\right)} = -\frac{1}{\theta}\boldsymbol{W}\left(\frac{e^{-1}\left(1 - a + \sqrt{a^2 - a + 1}\right)}{2a}\right) - \frac{1}{\theta} \quad (14)$$

Note: $Q_{\left(\frac{1}{2}\right)}$ is the median of the distribution.

3.3 Moments

Proposition 1: If a random variable X has a QTAD, the kth moment is obtained as:

$$E(X^k) = \frac{(k+1)!}{\theta^k}\left(1 - a + \frac{a}{2^{k+1}} + \frac{a(k+2)}{2^{k+2}}\right) \quad (15)$$

Proof:

$$E(X^k) = \int_0^\infty x^k f(x)\, dx = \int_0^\infty x^k \left(\theta^2 x e^{-\theta x} - a\theta^2 x e^{-\theta x} + 2a\theta^2 x e^{-2\theta x} + 2a\theta^3 x^2 e^{-2\theta x}\right) dx$$

$$= \theta^2 \int_0^\infty x^{k+1} e^{-\theta x} dx - a\theta^2 \int_0^\infty x^{k+1} e^{-\theta x} dx + 2a\theta^2 \int_0^\infty x^{k+1} e^{-2\theta x} dx + 2a\theta^3 \int_0^\infty x^{k+2} e^{-2\theta x} dx$$

$$= \frac{\theta^2}{\theta^{k+2}}\int_0^\infty u^{k+1}\, e^{-u} du - \frac{a\theta^2}{\theta^{k+2}}\int_0^\infty u^{k+1}\, e^{-u} du + \frac{2a\theta^2}{(2\theta)^{k+2}}\int_0^\infty v^{k+1}\, e^{-v} dv + \frac{2a\theta^3}{(2\theta)^{k+3}}\int_0^\infty v^{k+2}\, e^{-v} dv$$

Assuming $u = \theta x \;\rightarrow\; x = \frac{u}{\theta} \rightarrow dx = \frac{1}{\theta} du$ and $v = 2\theta x \;\rightarrow\; x = \frac{v}{2\theta} \rightarrow dx = \frac{1}{2\theta} dv$
Therefore,

$$E(x^k) = \frac{(k+1)!}{\theta^k} - \frac{a(k+1)!}{\theta^k} + \frac{a(k+1)!}{2^{k+1}\theta^k} + \frac{a(k+2)(k+1)!}{2^{k+2}\theta^k}$$
$$= \frac{(k+1)!}{\theta^k}\left(1 - a + \frac{a}{2^{k+1}} + \frac{a(k+2)}{2^{k+2}}\right)$$

3.4 Moment generating function

Proposition 2: If a random variable *X* has QTAD, the MGF is obtained as:

$$E(e^{tx}) = \frac{\theta^2}{(\theta-t)^2} - \frac{a\theta^2}{(\theta-t)^2} + \frac{2a\theta^2}{(2\theta-t)^2} + \frac{4a\theta^3}{(2\theta-t)^3} \tag{16}$$

Proof:

$$E(e^{tx}) = \int_0^\infty e^{tx} f(x)\, dx = \int_0^\infty e^{tx}\left(\theta^2 x e^{-\theta x} - a\theta^2 x e^{-\theta x} + 2a\theta^2 x e^{-2\theta x} + 2a\theta^3 x^2 e^{-2\theta x}\right) dx$$
$$= \theta^2 \int_0^\infty x e^{-(\theta-t)x}\, dx - a\theta^2 \int_0^\infty x e^{-(\theta-t)x}\, dx + 2a\theta^2 \int_0^\infty x e^{-(2\theta-t)x}\, dx + 2a\theta^3 \int_0^\infty x^2 e^{-(2\theta-t)x}\, dx$$

Assuming $u = (\theta - t)x \;\rightarrow\; x = \frac{u}{\theta-t} \rightarrow dx = \frac{1}{(\theta-t)} du$; $v = (2\theta - t)x \;\rightarrow\; x = \frac{v}{2\theta-t} \rightarrow dx = \frac{1}{(2\theta-t)} dv$
Therefore,

$$E(e^{tx}) = \frac{\theta^2}{(\theta-t)^2}\int_0^\infty u e^{-u}\, du - \frac{a\theta^2}{(\theta-t)^2}\int_0^\infty u e^{-u}\, du + \frac{2a\theta^2}{(2\theta-t)^2}\int_0^\infty v e^{-v}\, dv + \frac{2a\theta^3}{(2\theta-t)^3}\int_0^\infty v^2 e^{-v}\, dv$$
$$= \frac{\theta^2}{(\theta-t)^2} - \frac{a\theta^2}{(\theta-t)^2} + \frac{2a\theta^2}{(2\theta-t)^2} + \frac{4a\theta^3}{(2\theta-t)^3}$$

Hence, using either (15) or (16), the first four central moments of a random variable *X* with the QTAD are:

$$E(X) = \frac{8-3a}{4\theta} \tag{17}$$

$$E(X^2) = \frac{24-15a}{4\theta^2} \tag{18}$$

$$E(X^3) = \frac{96-75a}{4\theta^3} \tag{19}$$

$$E(X^4) = \frac{120-105a}{\theta^4} \tag{20}$$

3.5 Measures of variation

The variance of a random variable with QTAD us obtained as:

$$Var(X) = \frac{32-12a-9a^2}{16\theta^2} \tag{21}$$

Skewness and kurtosis of a distribution can be obtained from its central moments [14] respectively as:

$$SK(X) = \frac{E(X^3) - 3E(X^2)E(X) + 2\big(E(X)\big)^3}{\big(Var(X)\big)^{\frac{3}{2}}}$$

$$Kurt(X) = \frac{E(X) - 4E(X^3)E(X) + 6E(X^2)\big(E(X)\big)^2 - 3\big(E(X)\big)^4}{\big(Var(X)\big)^2}$$

Hence, skewness and kurtosis for a random variable X with QTAD are respectively obtained as:

$$SK(X) = \frac{54a^3+108a^2+48a-256}{(9a^2+12a-32)\sqrt{32-12a-9a^2}} \quad (22)$$

$$Kurt(X) = \frac{6144-2304a-2304a^2-648a^3-243a^4}{(9a^2+12a-32)^2} \quad (23)$$

The Dispersion Index is used to assess dispersion level in observations. A set of observations is dispersed when DI > 1; under-dispersed when DI < 1 and equi-dispersed when DI = 1. It is defined as:

$$DI(X) = \frac{Var(X)}{E(X)}$$

Hence, the Dispersion Index for a random variable X with QTAD is obtained as:

$$DI(X) = \frac{9a^2+12a-32}{4\theta(3a-8)} \quad (24)$$

Table 1. Estimates of different statistics of QTAD for different values of a and θ

	Variance			Dispersion Index			Skewness	Kurtosis
	θ=0.5	*θ=1.5*	*θ=10*	*θ=0.5*	*θ=1.5*	*θ=10*	*θ=0.50=1.5=10*	*θ=0.50=1.5=10*
a = – 0.9	8.88	0.99	0.02	1.66	0.55	0.08	1.19	5.29
a = – 0.5	8.94	0.99	0.02	1.88	0.63	0.09	1.22	5.31
a = – 0.1	8.28	0.92	0.02	1.99	0.66	0.10	1.36	5.79
a = 0.0	8.00	0.89	0.02	2.00	0.67	0.10	1.41	6.00
a = 0.1	7.68	0.85	0.02	1.99	0.66	0.10	1.47	6.25
a = 0.5	5.94	0.66	0.01	1.83	0.61	0.09	1.71	7.66
a = 0.9	3.48	0.39	0.01	1.31	0.44	0.07	1.66	8.13

Remarks:

i. For fixed θ, variance and dispersion index reduce as a increases.
ii. Since both skewness and kurtosis are independent of θ, their values are constant for different values of θ.
iii. As a increases, both skewness and kurtosis also increase.
iv. For fixed a, variance and dispersion index reduce as θ increases.

4 Parameter Estimation

If a random variable $X_i, i = 1, 2, 3, \ldots n$ comes from $QTAD \sim (\theta, a)$, then the likelihood function of X denoted by $\mathcal{L}$ is obtained as:

$$\mathcal{L} = \prod_{i=1}^{n} \left(\theta^2 x_i e^{-\theta x_i}\left(1 - a + 2ae^{-\theta x_i} + 2a\theta x_i e^{-\theta x_i}\right)\right)$$

Therefore, the log of the likelihood denoted with ℓ is obtained as:

$$\ell = 2n\log\theta + \sum_{i=1}^{n}\log(x_i) - \theta\sum_{i=1}^{n}x_i + \sum_{i=1}^{n}\log\left(1 - a + 2ae^{-\theta x_i} + 2a\theta x_i e^{-\theta x_i}\right)$$

The maximum likelihood estimates of (θ, a) denoted by $(\hat{\theta}, \hat{a})$ cannot be easily obtained by simple derivative since it forms a system of non-linear equations. The solutions are numerically obtained using **optimr** package in R language [15].

5 Applications

This section assesses the performance of the new proposition using referred lifetime data. Comparison of the new proposition (QTAD) and its special 1-parameter form are made with the exponential distribution, Ailamujia Distribution [6], and exponentiated Ailamujia distribution [16]. Table 2 shows the probability distribution functions of the compared models.

Table 2. Density functions of compared distributions

Distribution	pdf
Exponential	$\theta e^{-\theta x}$
Ailamujia	$\theta^2 x e^{-\theta x}$
Exponentiated Ailamujia	$a\theta^2 x e^{-\theta x}\left(1 - \theta x e^{-\theta x} - e^{-\theta x}\right)^{a-1}$
1-Parameter Transmuted Ailamujia	$2\theta^2 x e^{-2\theta x}(1 + \theta x)$
Quadratic Transmuted Ailamujia	$\theta^2 x e^{-\theta x}\left(1 - a + 2ae^{-\theta x} + 2a\theta x e^{-\theta x}\right)$

Four lifetime datasets (Table 3) that appeared in literature for model comparison are assessed. The first dataset represents the relief time (in minutes) of patients receiving analgesic. The data was first reported in [17] and also appeared in several propositions [16], [18–20] involving lifetime distributions. The second dataset consists of the running times and times of failures of sampled devices from a tracking study of a large system as reported in [21]. The third dataset is the maximum flood level (million cubic feet/second) in the Susquehanna River, Pennsylvania, USA first reported in [22] and later by several other authors [23,24] with propositions on different lifetime distributions. The fourth dataset is the waiting time (in minutes) of 100 bank customers before being served as used on the Lindley distribution by [25].

Table 3. Lifetime datasets

Dataset	Observations
I	1.1, 1.4, 1.3, 1.7, 1.9, 1.8, 1.6, 2.2, 1.7, 2.7, 4.1, 1.8, 1.5, 1.2, 1.4, 3.0, 1.7, 2.3, 1.6, 2
II	2.75, 0.13, 1.47, 0.23, 1.81, 0.30, 0.65, 0.10, 3.00, 1.73, 1.06, 3.00, 3.00, 2.12, 3.00, 3.00, 3.00, 0.02, 2.61, 2.93, 0.88, 2.47, 0.28, 1.43, 3.00, 0.23, 3.00, 0.80, 2.45, 2.66
III	0.654,0.613,0.315,0.449,0.297,0.402,0.379,0.423,0.379,0.324,0.269,0.740,0.418, 0.412,0.494,0.416,0.338,0.392,0.484,0.265
IV	0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11, 11, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27, 31.6, 33.1, 38.5

Table 4 presents some basic descriptive statistics of lifetime data assessed in the study. All datasets except dataset II are positively skewed. Datasets I and II are under-dispersed while datasets III and IV are over-dispersed.

Table 4. Descriptive Statistics for datasets assessed

Dataset	Median	Mean	Mode	Variance	Disp. Index	Skewness	Kurtosis
I	1.700	1.900	1.750	0.496	0.261	1.720	2.924
II	1.965	1.770	2.750	1.322	0.747	-0.284	-1.546
III	0.740	0.450	0.016	1.068	2.373	0.600	0.407
IV	8.100	9.877	7.500	52.374	5.303	1.473	2.540

Tables 5 – 8 show results for models comparisons. Four criteria (*-2 LL*, *AIC*, *BIC*, *and CAIC*) are used to compare performances of the distributions. Among the competing distributions, the one with the lowest value of each criterion provides the best fit for a particular dataset. The new proposition (Quadratic Transmuted Ailamuja Distribution, QTAD) provides the best fit for datasets I, II, and III with lowest values of all selection criteria. For the fourth dataset, QTAD has the least -2 LL, while the Ailamujia Distribution has the least values for AIC, BIC, and CAIC. Results also show that a special form of the QTAD with 1-paramter also gives better statistics in model fitting than both exponential and Ailamujia distributions (for dataset I, III, and IV).

Table 5. Estimates for data on relief time

Distribution	$\hat{\theta}$	$\hat{a}$	.-2LL	AIC	BIC	CAIC
Exponential	0.5263		65.6742	67.6742	68.6699	67.6742
Ailamujia	1.0526		52.3264	54.3264	55.3221	54.3264
Exponentiated Ailamujia	2.6406	13.2729	32.9900	36.9900	38.9815	37.2253
1-Parameter Transmuted Ailamujia	0.6699		50.4354	52.4354	53.4311	52.4354
Quadratic Transmuted Ailamujia	0.4384	5.2607	21.9792	25.9792	27.9707	26.2145

Table 6. Estimates for data on tracking of failure time

Distribution	$\hat{\theta}$	$\hat{a}$	.-2LL	AIC	BIC	CAIC
Exponential	0.5649		94.2701	96.2701	97.6713	96.2701
Ailamujia	1.1297		99.6531	101.6531	103.0543	101.6531
Exponentiated Ailamujia	0.8387	0.5710	93.0263	97.0263	99.8287	97.1745
1-Parameter Transmuted Ailamujia	0.6992		100.3149	102.3149	103.7161	102.3149
Quadratic Transmuted Ailamujia	0.4552	6.0730	53.2110	57.2110	60.0134	57.3591

Table 7. Estimates for dataset on flood

Distribution	$\hat{\theta}$	$\hat{a}$	.-2LL	AIC	BIC	CAIC
Exponential	2.3632		5.5989	7.5989	8.5946	7.5989
Ailamujia	4.7265		-8.3471	-6.3471	-5.3514	-6.3471
Exponentiated Ailamujia	12.9590	20.3410	-32.1679	-28.1679	-26.1764	-27.9326
1-Parameter Transmuted Ailamujia	3.0132		-10.4534	-8.4534	-7.4577	-8.4534
Quadratic Transmuted Ailamujia	1.9698	29.9543	-100.1443	-96.1443	-94.1528	-95.9090

Table 8. Estimates for data on customers' waiting times in bank

Distribution	$\hat{\theta}$	$\hat{a}$	.-2LL	AIC	BIC	CAIC
Exponential	0.1013		658.0418	660.0418	662.6470	660.0418
Ailamujia	0.2025		634.6014	636.6014	639.2066	636.6014
Exponentiated Ailamujia	0.2039	1.0138	634.5926	638.5926	643.8029	638.6338
1-Parameter Transmuted Ailamujia	0.1262		635.7438	637.7438	640.3490	637.7438
Quadratic Transmuted Ailamujia	0.1734	0.3818	634.3080	638.3080	643.5183	638.3492

6 Conclusions

In this study, we introduce a new 2-parameter lifetime distribution by transmuting the Ailamujia distribution [6] with quadratic transmutation map [2]. We explore shapes of the density functions and reliability functions and also present its various mathematical properties. Depending on the value of the scale parameter, the new proposition can efficiently model dispersed lifetime data. Comparison of the new proposed distribution with other related lifetime distributions reveal that it gives better fit in most cases.

Competing Interests

Author has declared that no competing interests exist.

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