



Analysis of Moving Chord Inclination Angles when Determining Curvature of Track Axis

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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ABSTRACT

The analysis presented in the paper explains computational issues related to the use of a new method of determining the curvature of the track axis – the so-called moving chord method. It indicates the versatility of this method – it may be used both in a horizontal and vertical plane. It also draws attention to its very high precision, as evidenced by the exemplary geometric cases under consideration. The focus here is on the computational foundations of the discussed method regarding the angles of inclination of the moving chord. It was found that for a circular arc in the horizontal plane, the inclination angles of the moving chord depend on the track turning angle, while the difference in inclination angles depends only on the radius of the arc. In the case of a circular arc in the vertical plane, the moving chord inclination angles are much smaller than in the horizontal plane, which is connected with the range of the applied radii of the arcs. As in the horizontal plane, the radius of the vertical curve is the only factor that determines the discrepancy in the inclination angles of the moving chord.

Keywords: Railway track; curvature of track axis; moving chord method; computational methodology.

1. INTRODUCTION

The achievable speed of trains is mainly determined by the shape of the track axis in the horizontal plane. Therefore, the key maintenance operation is to determine the basic geometric parameters: the location and length of straight sections, the location of circular arcs together with their radius and length, and the location of transition curves along with their type and length. The principles of carrying out appropriate measurements are similar across various railway authorities [1-7].

The traditional surveying methods that have been used for many years are very labour-intensive and, therefore, they require significant financial outlays. This situation was an inspiration to undertake research on the use of a new measurement technique, which was called the method of mobile satellite measurements [8-17]. This technology has been developed in Poland for over 10 years and since 2018 it has been applied in the current research project BRIK [18-19], the aim of which is to obtain a practical solution for implementation.

As a result of mobile satellite measurements, a set of numerical data is available, which, after appropriate processing, form a set of coordinates in the corresponding Cartesian system (in Poland – as far as the horizontal plane is concerned - the most common is the PL-2000 two-dimensional perpendicular coordinate system, which is an element of the national spatial reference system) [20]. Having a set of coordinates, one can focus on the issue of identifying the occurring geometrical elements. The method used so far has been based on a diagram of horizontal arrows from a chord

stretched along the path, which is the most frequently used tool for assigning track points to sections with defined geometry. Such a graph is identical with the horizontal arrow graph and the superelevation graph (if any). In practice, the arrow graph is often equated with the curvature diagram.

The article deals with selected specific issues related to the new method of determining the curvature of the track axis, the assumptions of which – for the horizontal plane – are described in [21] and supplemented in [22]. Namely, factors determining the curvature, i.e. the inclination angles of the moving chord, were analyzed, considering both the horizontal and the vertical curvature.

2. DETERMINING CURVATURE WITH THE APPLICATION OF THE MOVING CHORD METHOD

To define the curvature it is necessary to manipulate the angles of the tangent to the geometric system. This does not constitute a problem if you have an analytical record of a given curve. However, with the real railway track in mind, which is usually deformed as a result of operation, an idea appeared not to use the tangent but the corresponding chords when determining the curvature of the track. Paper [21] presents a method of changing inclination angles of a chord with a fixed length (the so-called moving chord method), of a theoretical character but verified on selected geometric systems. Using analytical notation, the position of the ends of the chord may be clearly determined in a given case. Fig. 1 shows a schematic diagram of determining the curvature with the application of the proposed method.

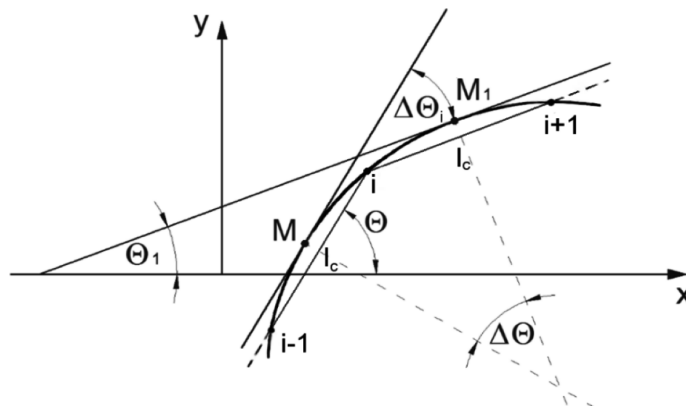


Fig. 1. Schematic diagram of determining the curvature by the moving chord method [21]

It was assumed that for the considered short sections of the track, the tangent and corresponding chords are parallel to each other, and the points of contact are projected perpendicularly onto the centre of the chord. Curvature k_i occurring at a given point i is determined by the following formula:

$$k_i = \frac{\Delta\Theta_i}{l_c} \quad (1)$$

where l_c is the length of the chord, and angle $\Delta\Theta_i$ results from the difference between the inclination angles of the adjacent chords converging at point i , i.e.

$$\Delta\Theta_i = \Theta_{i+(i+1)} - \Theta_{(i-1)+i} \quad (2)$$

where $\Theta_{i+(i+1)}$ is the inclination angle of the front chord, and $\Theta_{(i-1)+i}$ is the inclination angle of the rear chord.

The application of this procedure requires the knowledge of the coordinates of a given curve in the Cartesian system as angles $\Theta_{(i-1)+i}$ and $\Theta_{i+(i+1)}$ result from the inclination coefficients of the lines describing both chords.

As far as the sign of the curvature is concerned, the general rule is that if the curve turns to the right as the independent variable x increases, the curvature of the curve is negative, and if it turns to the left it is positive. This means that the positive value k_i in formula (1) corresponds with the case of convexity of the curve pointing downwards, and the negative value – with the one directed upwards.

Paper [21] presents a verification of the proposed method of determining the horizontal curvature on a clearly defined elementary geometric system of tracks, consisting of a circular arc and two symmetrically positioned transition curves (of the same type and length), calculated according to the principles of analytical design method [23]. Several geometric cases for various train speeds were considered, and the types of transition curves applied and the route turning angles were also differentiated. The obtained curvature diagrams were fully consistent with the diagrams constituting the basis for obtaining the corresponding geometric solution. This pertained both to the sections of the circular arc and the areas around transition curves.

It was also noted that the proposed method offers great opportunities of application. The practical aspect of the presented considerations may become evident when the geometrical characteristics of the track axis determined by measurements are not known and their determination becomes the primary goal. In this situation, the discussed method perfectly corresponds to the principles of mobile satellite measurements. Such measurements provide the coordinates of the track axis in the rectangular coordinate system in a very large number and in a very short time.

Two important detailed issues were discussed in [22] the influence of the chord length on the obtained values of horizontal curvature and the possibility of determining the location of border points between individual geometric elements. The analyzed variants depended on the type of transition curves used. It was found that the chord length within the range from 5 to 20 m does not play a significant role in determining the curvature and does not limit the application of the described method. We also note the precision of determining the nature of the curvature and its compliance with the theoretical course of transition curves. The analysis shows that in the moving chord method it is possible to determine the location of the border points between individual geometric elements, while the required chord length must be correlated with the type of transition curve. For example, for a transition curve in the form of a clothoid (with linear curvature), a chord with a length $l_c = 2$ m should be used locally, for a Bloss curve (i.e. a smooth curve), a chord with a length $l_c = 5$ m should be applied. In addition, some inaccuracies should be expected at both ends of clothoid (i.e. at the points between straight sections and the circular curve).

3. RESULTS AND DISCUSSION

3.1 Analysis of the Chord Inclination Angles in the Horizontal Plane

3.1.1 Principles of the conducted analysis

An analysis of the inclination angles of the moving chord in the horizontal plane was carried out in the x, y coordinate system, on the elementary geometric system of tracks, consisting of a circular arc with a 1000 m radius and two symmetrically positioned transition curves in the form of a 150 m long clothoid. The assumed superelevation $h_o = 105$ mm allows to

obtain train speed $V = 140$ km/h. When determining the speed of the train, the permissible values of kinematic parameters in force in Poland were taken into account: unbalanced acceleration on a circular arc $a_{per} = 0.85$ m/s² and acceleration increment on the transition curve $\psi_{per} = 0.3$ m/s³.

A universal mathematical notation of such a system was presented in [23]. Three special cases were considered, each one with a different turn angle: $\alpha = \pi/8$ rad, $\pi/4$ rad and $\pi/2$ rad, respectively.

The procedure consists of two main stages. First, the coordinates of successive points on the curve are determined, which are separated in a straight line by value l_c (i.e. by the chord length). As part of the analysis, $l_c = 5$ m was assumed and – due to the symmetry of the geometric system – the procedure was started from the point located in the centre of the system (i.e. on the circular arc), first covering the right side of the system. Then, on the basis of a mirror image, the required data for the left side of the system was completed.

In the second stage, the curvature of the track axis is determined. The main effort is focused on determining angle values $\theta_{(i-1)\div i}$ and $\theta_{i\div(i+1)}$. To do this, first determine the inclination coefficients of both adjoining chords using the following formulas:

$$s_{(i-1)\div i} = \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \quad (3)$$

$$s_{i\div(i+1)} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \quad (4)$$

As $\Theta_{(i-1)\div i} = \arctan s_{(i-1)\div i}$ and $\Theta_{i\div(i+1)} = \arctan s_{i\div(i+1)}$, we can apply formula (2) and determine the curvature with the application of formula (1).

3.1.2 Case 1 (track turning angle $\alpha = \pi/8$ rad)

Fig. 2 shows a diagram of the horizontal ordinates $y(x)$ obtained with the analytical design method [6] for the geometric system under consideration and the assumed track turn angle $\alpha = \pi/8$ rad. The same scale was applied to both the ordinate and abscissa axes.

As can be seen, at a small track turning angle $\alpha = \pi/8$ rad, the length of the entire geometric system is determined by the lengths of the transition curves (marked in blue). The ordinates $y(x)$ are relatively small; their maximum values (in the middle of the circular arc) slightly exceed 30 m.

Fig. 3 shows the corresponding diagram of the ordinates of curvature $k(l)$ determined by the moving chord method.

As was the case in [21-22], Fig. 3 shows complete compliance with the model solution – circular arc curvature is of constant value $k = 1/R$, and the change of curvature on transition curves in the form of clothoid is linear.

The curvature values result from the difference of the inclination angles of the chords derived from point i – backward $\theta_{(i-1)\div i}$ and forward $\theta_{i\div(i+1)}$. Fig. 4 shows diagrams of both these angles along the geometric system.

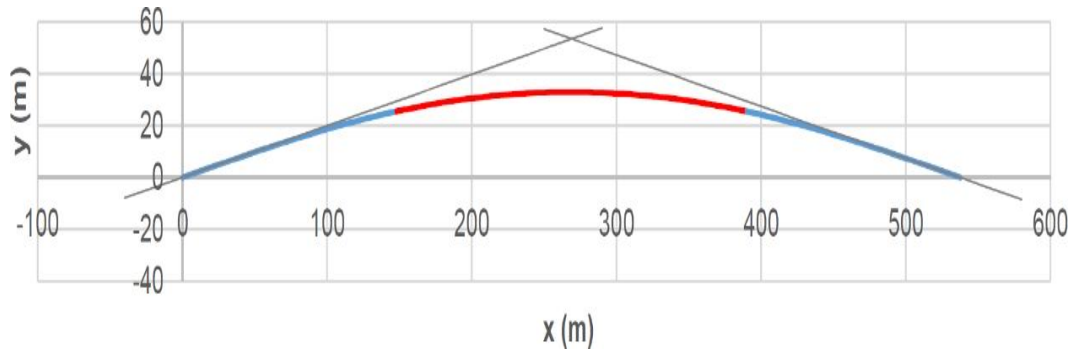


Fig. 2. Diagram of horizontal ordinates for the geometric system in Case 1 (circular arc in red, transition curves in blue; radius of circular arc $R = 1000$ m, transition curves in the form of clothoid with length $l_k = 150$ m and track turning angle $\alpha = \pi/8$ rad)

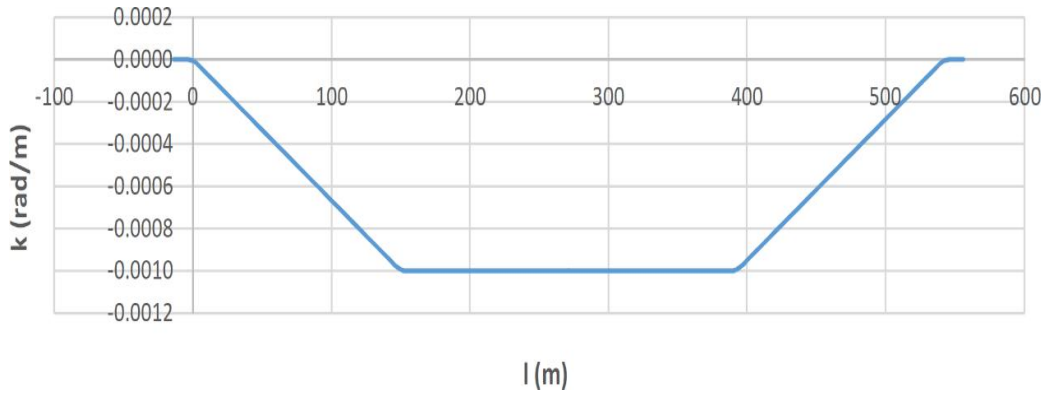


Fig. 3. Diagram of curvature along the entire length of the geometrical system in Case1 (circular arc radius $R = 1000$ m, transition curves in the form of clothoid with length $l_k = 150$ m and track turning angle $\alpha = \pi/8$ rad)

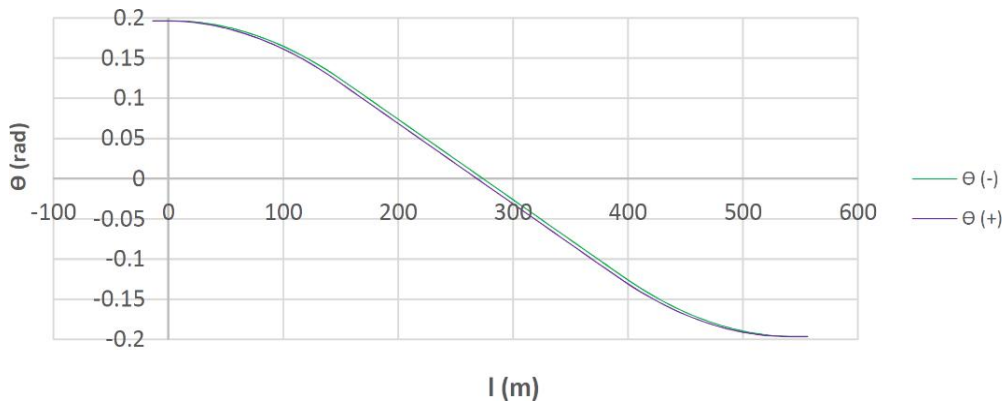


Fig. 4. Diagrams of the chord inclination derived from point i – backwards $\Theta (-)$ and forwards $\Theta (+)$ – in Case 1 (radius of circular arc $R = 1000$ m, transition curves in the form of clothoid with length $l_k = 150$ m and track turning angle $\alpha = \pi/8$ rad)

Fig. 4 clearly shows that along the circular arc there is a certain distance between the two lines (i.e. they are parallel to each other). By dividing the constant value of the difference of angles by the length of the chord, a constant value of curvature is obtained. Along the transition curves, the angle difference decreases to zero in the straight sections; in the case under consideration this leads to a linear curvature.

3.1.3 Case 2 (track turning angle $\alpha = \pi/4$ rad)

Fig. 5 presents a diagram of the horizontal ordinates $y(x)$ for the considered geometrical system and the assumed turning angle $\alpha = \pi/4$ rad. As in Case 1 (Fig. 2), the same scale was kept on the ordinate axis and the abscissa axis.

As can be seen, with the track turning angle twice as large as in Case 1, the length of the circular arc (marked in red) plays a decisive role

in the length of the entire geometric system. The system in Fig. 5 is almost twice as long as that in Fig. 2, and the ordinates $y(x)$ reach 100 m in the central section.

Fig. 6 shows the corresponding diagram of the ordinates of curvature $k(l)$ determined by the moving chord method. As can be seen, a significant extension of the circular arc section is the only difference between this and the situation in Fig. 3.

In this case, it would be difficult to draw the diagrams of chord inclination angles on an appropriate scale backward and forward from point i (as in Fig. 4) that would not be identical. Therefore, it was limited only to the extreme parts of the geometric system. Fig. 7 shows the graphs of angles $\theta_{(i-1)\div i}$ and $\theta_{i\div(i+1)}$ for the initial section, and Fig. 8 – for the final one.

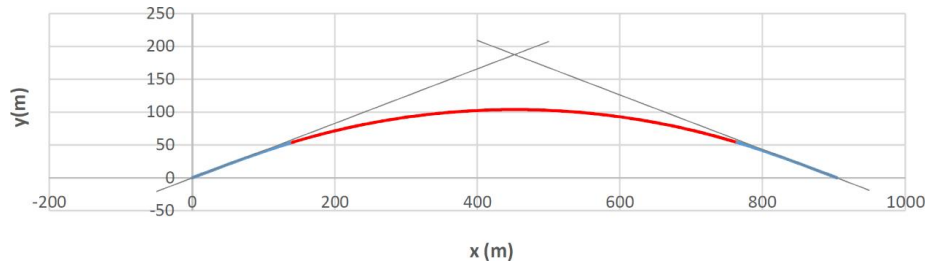


Fig. 5. Diagram of horizontal ordinates for the geometric system in Case 2 (circular arc in red, transition curves in blue; radius of circular arc $R = 1000$ m, transition curves in the form of clothoid with length $l_k = 150$ m and track turning angle $\alpha = \pi/4$ rad)

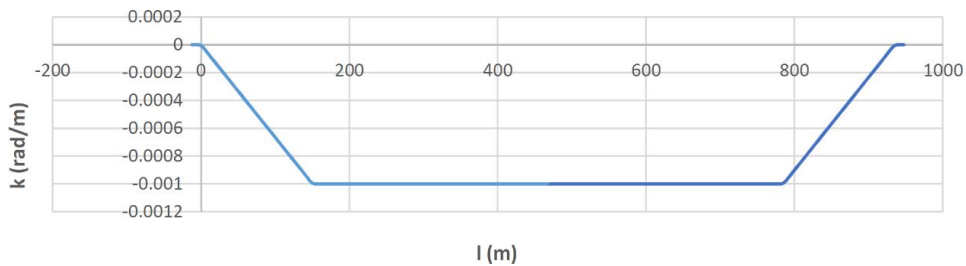


Fig. 6. Diagram of curvature along the geometrical system in Case 2 (radius of circular arc $R = 1000$ m, transition curves in the form of clothoid with length $l_k = 150$ m and track turning angle $\alpha = \pi/4$ rad)

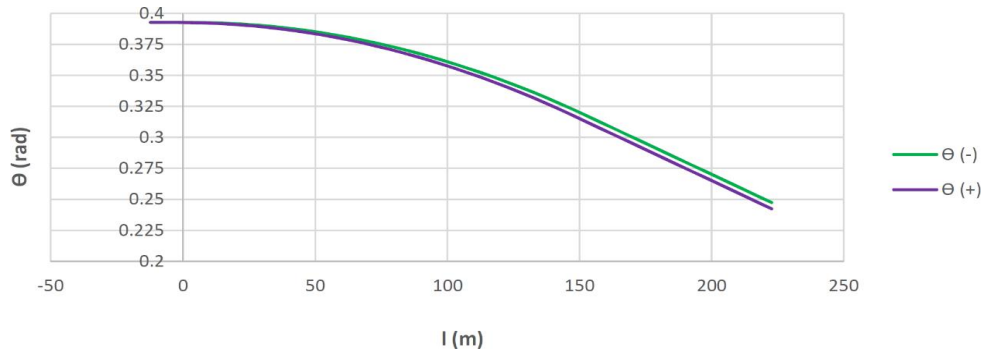


Fig. 7. Diagrams of the inclination of the chords derived from point i – backwards $\Theta (-)$ and forwards $\Theta (+)$ – for the initial section in Case 2 (radius of circular arc $R = 1000$ m, transition curves in the form of clothoid with length $l_k = 150$ m and the track turn angle $\alpha = \pi/4$ rad)

The graphs in Fig. 7 and 8 fully confirm the observations on Case 1 (Fig. 4). A stable distance is maintained between the two lines over the circular arc, and in the transition curves the difference between the angles decreases to zero on adjacent straight lines.

3.1.4 Case 3 (track turning angle $\alpha = \pi/2$ rad)

Fig. 9 demonstrates a diagram of the horizontal ordinates $y(x)$ for the considered geometrical system and the assumed track turning angle $\alpha =$

$\pi/2$ rad. As in Cases 1 and 2 (Figs. 2 and 5), the same scale was applied to the ordinate axis and the abscissa axis.

As can be seen, with the turning angle $\alpha = \pi/2$ rad for the entire length of the geometrical system, what is a decisive factor is the length of the circular arc (marked in red). The layout in Fig. 9 is almost three times the length of the layout in Fig. 2 and more than 50% longer than the layout in Fig. 5; ordinates $y(x)$ reach maximum values of about 350 m in the central section.

Fig. 10 shows the corresponding diagram of the ordinates of curvature $k(l)$ determined by the moving chord method. As can be seen, compared to the situation in Figs 3 and 6, the lengths of the transition curves here represent only a small percentage of the entire system.

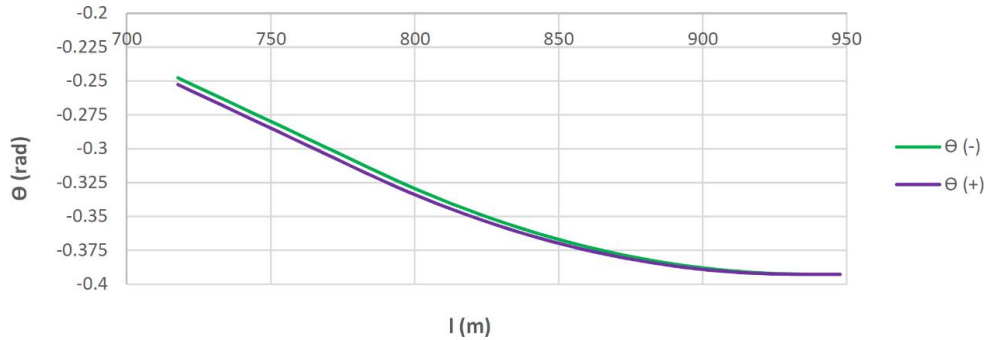


Fig. 8. Diagrams of the inclination angles of the chords derived from point i – backwards $\Theta (-)$ and forwards $\Theta (+)$ – for the end section in Case 2 (radius of circular arc $R = 1000$ m, transition curves in the form of clothoid with length $l_k = 150$ m and the track turn angle $\alpha = \pi/4$ rad)

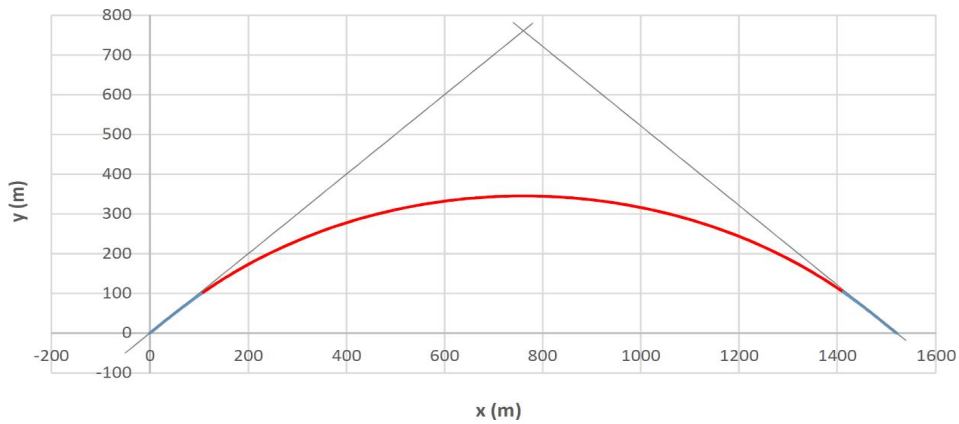


Fig. 9. Diagram of horizontal ordinates for the geometric system in Case 3 (circular arc in red, transition curves in blue; radius of circular arc $R = 1000$ m, transition curves in the form of clothoid with length $l_k = 150$ m and track turning angle $\alpha = \pi/2$ rad)

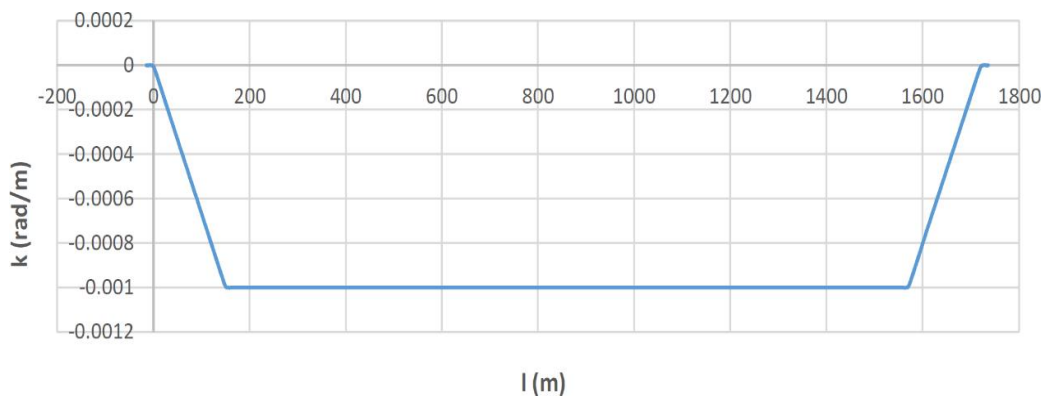


Fig. 10. Diagram of curvature along the geometrical system in Case 3 (radius of circular arc $R = 1000$ m, transition curves in the form of clothoid with length $l_k = 150$ m and track turning angle $\alpha = \pi/2$ rad)

In the case under consideration, the diagrams of angles $\theta_{(i-1)\div i}$ and $\theta_{i\div(i+1)}$ were neglected, as they would basically coincide with each other (even in the extreme sections). Simultaneously, the corresponding numerical values were taken into account in the further analysis carried out in section 3.1.5.

3.1.5 Comparison of inclination angle values

Table 1 presents a list of selected values of the inclination angles of the moving chord when determining the curvature of the track axis in Cases 1 ÷ 3, i.e. for the track turning angle $\alpha = \pi/8$ rad, $\pi/4$ rad and $\pi/2$ rad. These values pertain to the area of transition from the circular arc to the transition curve (the linear coordinate l_{EA} of the connection point of these geometric elements is given).

Table 1 clearly shows that for a given radius of a circular arc, the track turning angle determines the moving chord inclination angles – they are the greater, the greater α is. In the considered cases, the order of magnitude of the angles $\theta_{(i-1)\div i}$ and $\theta_{i\div(i+1)}$ for individual α differs significantly (from 0.1 to 0.7 rad), yet the differences of these angles are the same. On a circular arc they are 0.005 rad, given a curvature of 0.001 rad/m. The data in Table 1 provide a numerical illustration of the moving chord method, indicating its very high precision.

3.2 Analysis of Chord Inclination Angles in Vertical Plane

The geometric presentation of the track in the vertical plane (i.e. the longitudinal profile of the track) is presented in the rectangular coordinate system l, h , where the abscissa l represents the longitudinal dimension of the track axis, and the ordinate h is the projection of the track axis onto

the vertical plane. This system includes elements appearing on the horizontal plane – there are straight sections with a uniform inclination and circular arcs rounding the bends of the longitudinal profile. Thus, from a formal point of view, there is nothing to prevent the moving chord method from being used to determine the vertical curvature. However, the specificity of such a geometric system should be taken into account: the occurring longitudinal inclinations are very small (most often they amount to a few per mille), the bend values can not exceed the established allowed values, and the radii of the vertical curves must be far larger than the radii of the horizontal curves (even by an order of magnitude). As a result, the lengths of the vertical arc sections are smaller than their counterparts in the horizontal plane.

The analysis of the inclination angles of the moving chord in the vertical plane was carried out on a geometric system consisting of two symmetrically positioned sections with a uniform inclination equal to 2,5‰ and a circular arc with a radius of 10,000 m. Fig. 11 shows the plot of height ordinates $h(l)$ for the considered system. As these ordinates take very small values (3 cm max), the graph was drawn up with the axes representing two different scales.

In determining the vertical curvature, the abscissa values for successive points in the curve $h(l)$, spaced apart by the value l_c , are taken along axis l ; they are the result of the following dependence: $l_{i+1} = l_i + l_c$. As in the case of the horizontal plane, due to the symmetry of the geometric system, proceedings were started from a point located in the centre of the system (i.e. on the circular arc), first covering the right side of the system. Then, like in a mirror image, the required data for the left side of the system were completed.

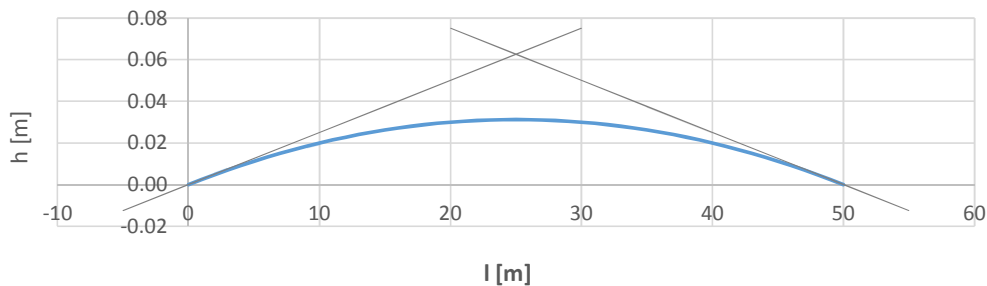


Fig. 11. Graph of elevation ordinates along a geometric system consisting of two sections with a uniform inclination equal to 2,5‰ and a circular arc with a 10,000 m radius (where both axes represent two different scales)

Table 1. List of selected values of the inclination angles of the moving chord when determining the curvature of the track axis in Cases 1 ÷ 3

Turn angle α (rad)	Location of end of arc l_{EA} (m)	Location of point l_i (m)	Angle $\theta_{(i-1)\div i}$ (rad)	Angle $\theta_{i\div(i+1)}$ (rad)	Angle difference $\Delta\theta_i$ (rad)	Curvature k_i (rad/m)
$\pi/8$	392.699082	371.349645	-0.097500	-0.102500	-0.005000	-0.001000
		376.349650	-0.102500	-0.107500	-0.005000	-0.001000
		381.349655	-0.107500	-0.112500	-0.005000	-0.001000
		386.349661	-0.112500	-0.117500	-0.005000	-0.001000
		391.349666	-0.117500	-0.122489	-0.004989	-0.000998
		396.349671	-0.122489	-0.127367	-0.004877	-0.000976
		401.249676	-0.127367	-0.132079	-0.004712	-0.000942
		406.349680	-0.132079	-0.136624	-0.004545	-0.000909
		411.349684	-0.136624	-0.141002	-0.004378	-0.000876
$\pi/4$	785.398163	416.349688	-0.141002	-0.145214	-0.004212	-0.000842
		762.699389	-0.292500	-0.297500	-0.005000	-0.001000
		767.699394	-0.297500	-0.302500	-0.005000	-0.001000
		772.699399	-0.302500	-0.307500	-0.005000	-0.001000
		777.699405	-0.307500	-0.312500	-0.005000	-0.001000
		782.699410	-0.312500	-0.317498	-0.004997	-0.000999
		787.699415	-0.317498	-0.322417	-0.004919	-0.000984
		792.699420	-0.322417	-0.327173	-0.004757	-0.000951
		797.699424	-0.327173	-0.331763	-0.004590	-0.000918
$\pi/2$	1570.796327	802.699429	-0.331763	-0.336186	-0.004423	-0.000885
		807.699433	-0.336186	-0.340443	-0.004257	-0.000851
		1550.398882	-0.687501	-0.692501	-0.005000	-0.001000
		1555.398887	-0.692501	-0.697501	-0.005000	-0.001000
		1560.288893	-0.697501	-0.702501	-0.005000	-0.001000
		1565.398898	-0.702501	-0.707501	-0.005000	-0.001000
		1570.398903	-0.707501	-0.712479	-0.004978	-0.000996
		1575.398908	-0.712479	-0.717326	-0.004847	-0.000969
		1580.398913	-0.717326	-0.722006	-0.004680	-0.000936
1585.398917	-0.722006	-0.726519	-0.004513	-0.000903		
1590.398921	-0.726519	-0.730865	-0.004347	-0.000869		
1595.398925	-0.730865	-0.735045	-0.004180	-0.000836		

Having the coordinates of individual points which mark the ends of the moving chord, for each point i the inclination coefficients of both adjoining chords were determined, using the following formulas:

$$s_{(i-1)\div i}^v = \frac{h_i - h_{i-1}}{l_c} \quad (5)$$

$$s_{i\div(i+1)}^v = \frac{h_{i+1} - h_i}{l_c} \quad (6)$$

Equations (5) and (6) give the values of the respective inclination angles: $\theta_{(i-1)\div i}^v = \text{atan } s_{(i-1)\div i}^v$ and $\theta_{i\div(i+1)}^v = \text{atan } s_{i\div(i+1)}^v$. The vertical curvature k_i^v is determined from the formula

$$k_i^v = \frac{\theta_{i\div(i+1)}^v - \theta_{(i-1)\div i}^v}{l_c} \quad (7)$$

Fig. 12 shows the obtained graph of the vertical curvature ordinates $k_i^v(l)$ determined by the moving chord method with a length of $l_c = 5$ m.

As for the horizontal plane, in the central part of Fig. 12 (i.e. on the circular arc) there is complete compliance with the model solution – the

curvature of the circular arc is a constant value, equal to 0.0001 rad/m. In the extreme sections there is a variation in curvature, which is undoubtedly related to the length of the applied chord (the curvature is determined by three chords). Therefore, appropriate calculations were also carried out for a 2 m long chord (Fig. 13), resulting in a radical shortening of transition zones.

The values of the vertical curvature are a result of the difference in the angles of the chord inclination $\theta_{(i-1)\div i}^v$ and $\theta_{i\div(i+1)}^v$, starting from point i . Fig. 14 shows the graphs of these angles along the geometrical system for chord length $l_c = 5$ m.

As in the case of the horizontal plane, the distance between the two diagrams is maintained along the circular arc (it drops to zero at the extreme sections). Dividing the constant angle difference of 0.0005 rad by the chord length, we obtain constant value of the curvature of the vertical curve $k = 0.0001$ rad/m. The difference between the angles in the horizontal plane for the same chord length (Figs. 4, 7 and 8 and Table 1) was 0.0005 rad on the circular arc.

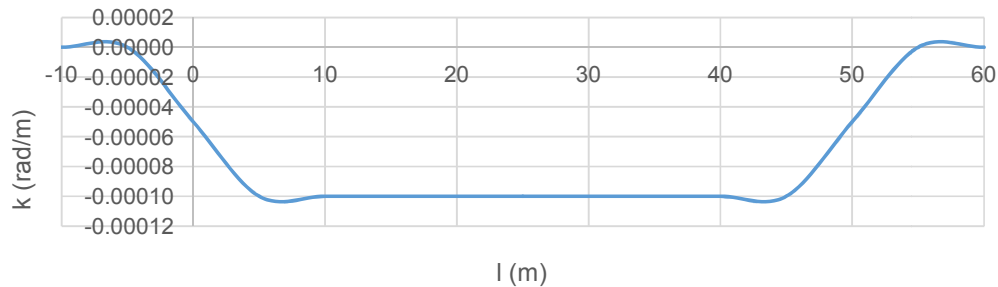


Fig. 12. Graph of vertical curvature along the geometric system consisting of two sections with a uniform inclination equal to 2,5‰ and the circular arc with a radius of 10,000 m (chord length $l_c = 5$ m)

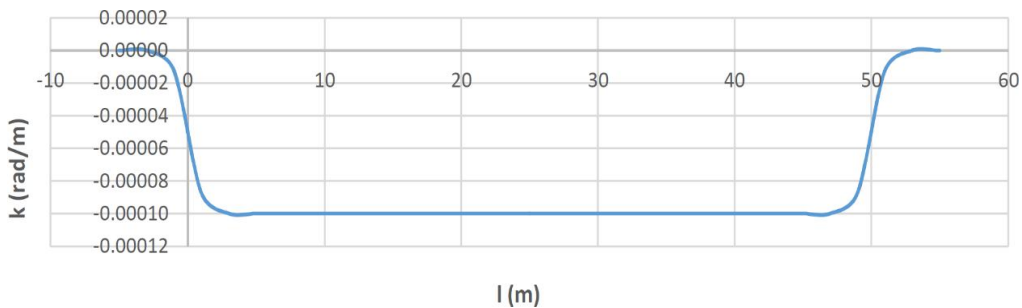


Fig. 13. Graph of vertical curvature along the geometric system consisting of two sections with a uniform inclination equal to 2,5‰ and the circular arc with a radius of 10,000 m (chord length $l_c = 2$ m)

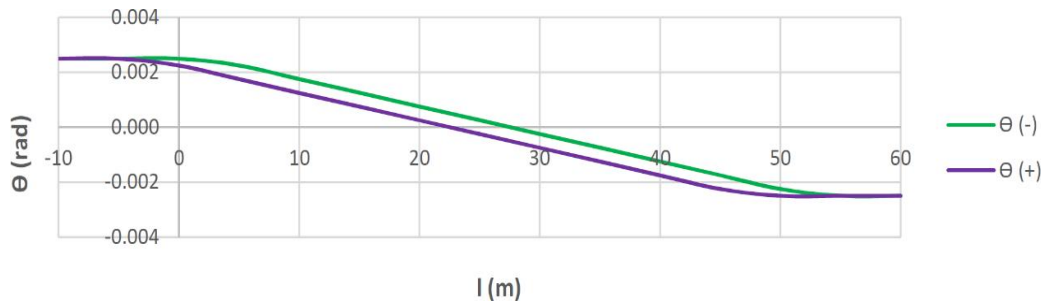


Fig. 14. Graphs of the chord inclination in the vertical plane derived from point i – backwards Θ (-) and forward Θ (+) – for $l_c = 5$ m (two symmetrically positioned sections with a uniform inclination equal to 2.5 ‰, circular arc radius $R = 10,000$ m)

A similar diagram as shown in Fig. 14 applies for the chord length $l_c = 2$ m, but the difference in the inclination angles of chords $\theta_{(i-1)\div i}^v$ and $\theta_{i\div(i+1)}^v$ is smaller, equal to 0.0002 rad. Dividing this value by the length of the chord we obtain the same curvature of the circular arc as before.

4. CONCLUSION

Years of search for an effective method of identifying geometric systems of the railway track have resulted in the development of a new method of determining the curvature of the track axis. Its essence is operating the inclination angles of a fixed length chord in a Cartesian coordinate system. The assumptions of the so-called moving chord method were presented in [21], where it was also verified on selected model geometric systems. Both the above-mentioned study and the supplementary article [22] focused on the issue of horizontal curvature.

This study considers the problem of determining the curvature both in the horizontal plane and in the vertical plane. The focus is on the computational basis of the discussed method, concerning the angles of inclination of the moving chord. By dividing the angle difference at a given point by the length of the chord the desired value of the curvature is obtained.

It was found that for a circular arc in the horizontal plane, the inclination angles of the moving chord depend on the track turning angle (they are the greater, the greater the angle α is). However, the difference in the inclination angles depends only on the circular arc radius. In the case of a circular arc in the vertical plane, the inclination angles of the moving chord are much smaller than in the horizontal plane. It is connected with the range of the arc radii. The radius of the vertical curve is the only factor

determining the difference in the moving chord angles.

The analysis presented in the paper explains the computational issues related to the use of the new method of determining the curvature of the track axis and indicates the universality of this method as it can be applied both to the horizontal and vertical planes. It also draws attention to its very high precision, as evidenced by the exemplary geometric cases under consideration.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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