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An Improved Two-states Cyclical Dynamic Model for Plastic Waste Management

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Authors' contributions

This work was carried out in collaboration among all authors. The need to review was conceived by all authors. Author JAA designed the proposed improvement. The revision of the proposed design was done by Authors ENW and FIA. Author JAA searched for literature and analytically solved the system. The R-based programme was written by authors JAA and ENW. The Analysis of the work was conducted by authors ENW and FIA. Author JAA wrote the initial draft of the manuscript but the final revision was executed by authors ENW and FIA. All authors read and approved the final manuscript.

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Abstract

The panacea to the global challenge of plastic waste management is the transition towards plastic circular economy, which can be sustained through tailor-made management strategies. However, cutting-edge strategic solutions are constrained by inadequate data due to inadequate plastic-based predictive models. This paper presents an improved version of an existing two-state cyclical dynamic closed (CDC) model. The CDC model was formulated using a homogeneous linear system of ordinary differential equations (ODEs) and was modified by introducing a separation target which plays an essential role in determining both quantity and quality of recycled plastics. The Laplace transforms technique was the main analytic solution technique used. Values of the parameters were computed using the global plastic data applied for the existing CDC model, and with a technique termed the *nth-order product derivative proximity*, alternating pairs of initial values were selected each for the global annual plastic production and the global annual plastic waste generation. The validation process of the new CDC model was accomplished using the root mean squared error (RMSE)

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and the mean average percentage error (MAPE), which are measures of the model's predictive power. Comparatively, RMSEs of the new CDC model were smaller than the RMSEs of the existing CDC model. MAPEs for the new CDC model were 6.5% and 7% (as against 13% and 18% in the existing model) respectively for the global annual plastic: production and waste generation, indicating that the new model predicts with 93.5% and 93% degrees of accuracy respectively for the global annual plastic: production and waste generation. Therefore, the new CDC model has outperformed the existing CDC model in terms of predictive power, and thus, establishing the new CDC model as an improved version of the existing one. The model can therefore make impactful policy decisions for sustainable plastic waste management thereby aiding to achieve the transition towards circular economy in plastic waste management.

Keywords: Separation target; nth-order product derivative proximity; cyclical dynamics; closed model.

1 Introduction

The challenge of plastic waste management has still remain a growing worldwide concern. The excruciating nature of the problem fueled by lack of accurate or reliable data to engineer optimal planning, decision - making and policy formulation. The incident of inadequate and unreliable data was opined to have emanated from the existence of very few predictive models in the area of plastic waste management [1], and suggestively, the inadequate quantitative treatment of issues concerning plastic waste management [2]. Because the area of plastic waste and waste management in general is dominated with qualitative approach, quantification of crucial correlates of unsustainable plastic waste management has remain a ubiquitous challenge. It was recently revealed [1] that the very few predictive models do not reflect the roles of crucial determinants such as the recycling rate, waste generation rate, incineration and discarding rates. In fact, most of the studies on life-cycle assessment of plastics do not exemplify the complete dynamic variation of the plastic life-cycle (PLC). The complete dynamic of the PLC involves both forward and reverse logistics. Direct determinants of the forward logistics activities include primary plastic production rate, waste generation rate, among others, while the recycling rate, incineration and discarding rates constitute reverse logistics activities. Hence, predictive accuracy of models will be affected if these significant determinants are excluded in modelling predictive and forecasting models for plastic production and its corresponding waste generation. We contend that a good forecasting model with a potential predictive accuracy will be essential for optimal decision-making and policy formulation for efficient management of plastic waste. Addor et al. [1] developed a cyclical dynamic closed (CDC) model involving two states for plastic waste management, which reflected the roles of the recycling rate, waste generation rate, incineration rate and the discarding rate. Although the model exhibited a higher degree of predictive or forecasting accuracy, that is 87 percent (for the plastic production model) and 83 percent (for the plastic waste generation model), the model did not account for the role of plastic waste separation. The waste separation rate is very crucial in plastic waste recycling since the plastics waste streams are contaminated with varying degrees of impurities [2, 3, 4, 5, 6, 7, 8, 9]. The quality and quantity of recycled plastics are affected by the level of separation [10,11,12] and every decision-making involving this parameter is pareto-exemplified, since there are trade-off implications of choosing between high and low targets of separation. High level of separation will imply high quality of both recycled and recovered waste, and vice versa. Therefore, the existing CDC model proposed does not represent the complete dynamic complexities of plastic waste management. Hence, the aim in this paper is to improve upon the above CDC model by introducing a separation target. The motivation behind this study was engineered by the limited application of time-dependent deterministic or predictive models to the study of the dynamics of the PLC.

Contribution to science: The study contributes to science by improving the predictive accuracy of the existing CDC model for plastic waste management. The improvement was achieved by the introduction of the plastic waste separation target which cannot be neglected in modelling the dynamics of the PLC. Thus, introducing this parameter extends the existing CDC model in terms of its representation of a real-life exposition of plastic waste management. Another factor that helped to improve predictive accuracy was the proposal of a new technique of selecting and alternating pairs of initial values, which we have labelled the *nth-order product derivative proximity*. The improvement in the predictive accuracy will therefore, help to optimize decision-making and policy formulation for effective, efficient and sustainable plastic waste management.

2 Materials and Methods

2.1 Model development

In this section, the main task is to adopt and modify to improve the cyclical dynamic closed (CDC) model that was developed by [1]. The assumptions underlying the original CDC model are as follows:

- The plastic wastes management is considered within the frame work of a closed system, where no newvirgin plastics are produced. The motive is to ensure that plastic production occurs only through recycling, this will help to ascertain if plastic wastes management can be sustained only through recycling.
- The plastic recycling and waste generation rates occur according to a Poisson process. This is to help to ascribe randomness to the plastic recycling and plastic wastes generation processes so that transitional probabilities can be derived.
- Technology is fixed at a value 1 (Cobb-Douglas production Function)
- The role of a plastic waste receptor is ignored
- The role of plastic waste separation is ignored.

We denote by $x(t)$, the volume of plastic products at the consumption unit at any given point in time (t); $y(t)$, the volume of plastic wastes at by the production unit at any given point in time (t); μ, plastic waste generation rate; ψ, plastic recycling rate and; w the combined rate of plastic waste incineration rate (w_i) and discard (w_d), where $w = w_i + w_d$. It is assumed that with these variables and their respective rates, the volume of plastic waste that will be generated at the household or consumption unit to be transferred to the production unit at any given point in time is $\boldsymbol{\mu x}$. Also, total volume of recycled plastics at any given point in time is $\boldsymbol{\psi y}$. All plastic waste generated at the household are transferred to the production unit, therefore, the volume of plastic products experiences a decrement of $\boldsymbol{\mu x}$. As a result, the volume of plastic waste at the production unit faces an increment of $\boldsymbol{\mu x}$. Similarly, since all recycled plastic products are sent to the household for consumption, the volume of plastic products at the household faces an increment of $\boldsymbol{\psi y}$**,** while the volume of plastic waste at production unit experiences a decrement of $\boldsymbol{\psi y}$. In addition, the amount of plastic waste faces another source of decrement from the w to the environment or landfills. Hence, the total decrement to plastic waste at the production unit due to incineration and discarding of plastic waste at any point in time (t) is wy.

Based on the above descriptions and explanations, the guiding equations for original CDC model developed by [1] is given by the system of homogeneous linear ordinary differential equations below.

$$\left.\begin{aligned} \frac{dy(t)}{dt} &= \mu x - (\psi + w)y \\ \frac{dx(t)}{dt} &= \psi y - \mu x \end{aligned}\right\} \tag{1}$$

The above equation, (1) represents the CDC non-separation model for a 2-multistate cyclical dynamics of plastic wastes management. Neglecting the role of waste separation in the original CDC model for plastic waste management is not a realistic assumption since separation plays a pivotal role in determining both quantity and quality of recovered plastic wastes as well as recycled plastics. Although the model outperformed other equally best performing models, we still believe that integrating the role of the separation target will still improve the model. By assuming the role of plastic waste separation, which is innovatively motivated at a separation target $\boldsymbol{\beta}$**,** our modified version of the CDC model for plastic waste management can be specified by Equation 2.

$$\left.\begin{aligned} \frac{dy(t)}{dt} &= \mu(1-\beta)x - (\psi + w)y \\ \frac{dx(t)}{dt} &= \psi y - \mu(1-\beta)x \end{aligned}\right\} \tag{2}$$

By the original concept, $\boldsymbol{\beta}$ is supposed to be a decrement to both x and y. However, technological innovation is introduced so that the amount of separation can be recovered as a product. Therefore, in this version, the separation target is an increment to plastic products but a decrement to plastic waste at the production unit.

Laplace transforms (LT) is the solution technique applied in solving the system represented by Equation 2. Following the LT technique, we obtain as follows:

$$\ell\left(\frac{dy(t)}{dt}\right) = \ell[\mu(1-\beta)x - (\psi + w)y]$$

$$sY(s) - y(0) = \mu(1-\beta)X(s) - \psi Y(s)$$

By applying the initial conditions, we simplify to get

$$Y(s)(s+\psi) = \mu(1-\beta)X(s) + y_0$$
$$\Rightarrow Y(s) = \frac{\mu(1-\beta)X(s) + y_0}{s+\alpha}. \tag{3}$$

Where $\alpha = \psi + w$

Following similar approach, we have

$$\ell\left(\frac{dx(t)}{dt}\right) = \ell[\psi y - \mu(1-\beta)x]$$
$$\Rightarrow X(s) = \frac{\psi Y(s) + x_0}{s + \mu(1-\beta)} \tag{4}$$

If Equation 3 is substituted into Equation 4, we have

$$X(s) = \frac{\psi\left[\frac{\mu(1-\beta)X(s)+y_0}{s+\alpha}\right] + x_0}{s+\mu(1-\beta)},$$

Which is simplified to

$$X(s) = \frac{\psi y_0 + x_0(s+\psi+w)}{\left(s+\frac{k_1}{2}\right)^2 - \left[\frac{\sqrt{{k_1}^2 - 4kw}}{2}\right]^2}$$

Where $k = \mu(1-\beta), k_1 = k + \psi + w$

If we decompose the $X(s)$ into partial fractions, we have

$$\frac{\psi y_0 + x_0(s+\psi+w)}{(s+K)^2 - \lambda^2} = \frac{M+Ns}{(s+K)^2 + \lambda^2},$$
$$\Rightarrow \psi y_0 + x_0(s+\psi+w) = M + Ns.$$

Where,

$$M = \psi(y_0 + x_0) + wx_0, N = x_0, \lambda = \frac{\sqrt{{k_1}^2 - 4kw}}{2}, K = \frac{k_1}{2}$$

By simplifying and applying the inverse Laplace transform, the solution $x(t)$ is given by

$$x(t) = \exp(-Kt)\left\{x_0\left[\cosh(\lambda t) - \frac{K}{\lambda}\sin(\lambda t)\right] + K_1 \sin h(\lambda t)\right\},$$
$$x(t) = \exp(-Kt)[x_0 \cos h(\lambda t) + K_2 \sin h(\lambda t)], \tag{5}$$

where,

$$K_1 = \frac{\psi(y_0 + x_0) + wx_0}{\lambda}, K_2 = \frac{\lambda K_1 - x_0 K}{\lambda}$$

Similarly, we solve for $y(t)$ by substituting (3) into (4) as follows:

$$Y(s) = \frac{\mu(1-\beta)\left[\frac{\psi Y(s) + x_0}{s + \mu(1-\beta)}\right] + y_0}{s + \psi + w}$$

Simplifying and denoting the following parameters;

$$k = \mu(1-\beta), k_1 = k + \psi + w, K = \frac{k_1}{2}, \lambda = \frac{\sqrt{{k_1}^2 - 4kw}}{2}, M = k_1(x_0 + y_0), N = y_0$$

$Y(s)$ is decompose into partial fractions

$$Y(s) = \frac{y_0 s + k_1(x_0 + y_0)}{(s+K)^2 - \lambda^2} = \frac{M + Ns}{(s+K)^2 - \lambda^2}.$$

The inverse Laplace transform is applied as follows, after simplifying we have $y(t)$ given by

$$y(t) = \exp(-Kt)[y_0 \cos h(\lambda t) + K_4 \sin h(\lambda t)], \tag{6}$$

where,

$$K_3 = \frac{k_1(x_0 + y_0)}{\lambda}, K_4 = \frac{\lambda K_3 - y_0 K}{\lambda}$$

The solutions, Equations 5 and 6, represent the predictive models for plastic production and plastic waste generation. Hence, the modified CDC model is represented by Equations 5 and 6. It is important to note that the steady state solution is zero, therefore, introducing the separation target does not change the steady state solution as obtained in the original CDC model.

2.2 Fitting the model to real data

Data and computations of values of parameters.

The values of the parameters ψ, μ and β, will be computed. For the purpose of comparison, we applied the same data as used in [1]. The parameters were computed using the following equations:

$$\psi = \left(\frac{\sum_{n=0}^{34} {x_n}^{rec}}{\sum_{n=0}^{34} y_n} \right) \tag{7}$$

$$\mu = \left(\frac{\sum_{n=0}^{34} y_n}{\sum_{n=0}^{34} x_n} \right). \tag{8}$$

$$w_i = \frac{\sum_{n=0}^{34} I_n}{\sum_{n=0}^{34} y_n} \quad . \tag{9}$$

Where, we denote in addition; the number of years (n), the volume of incinerated plastic wastes (I), the volume of recycled wastes (x^{rec}), and the volume of discarded wastes (y_d). Applying the data in Table 1 to the Equations 7 to 9, we have the computed values of the parameters.

$\psi = 0.146419796$
$\mu = 0.814742289$
$w_i = 0.206419796$
$w_d = 0.647160408$
$w = w_i + d_d = 0.853580204$

Table 1. Global annual volume of production, waste generation and recycled plastics

Year	Production	Waste Generation	Recycled
1988	110000000	41731536.85	250389.2211
1989	114000000	45491548.32	591390.1282
1990	120000000	49635828.37	992716.5675
1991	124000000	54152688.76	1462122.596
1992	132000000	59031846.01	2007082.764
1993	137000000	64126294.32	2629178.067
1994	151000000	69602679.86	3340928.633
1995	156000000	75560669.26	4155836.809
1996	168000000	82361129.49	5106390.028
1997	180000000	89683033.9	6188129.339
1998	188000000	97584109.19	7709144.626
1999	202000000	106376437.4	8829244.306
2000	213000000	116163069.7	10454676.27
2001	218000000	126733909	12293189.17
2002	231000000	138013226.9	14353375.6
2003	241000000	150434417.3	16698220.32
2004	256000000	163973514.9	19348874.76
2005	263000000	178731131.2	22341391.4
2006	280000000	194816933	25715835.16
2007	295000000	212350457	29516713.53
2008	281000000	231461998.1	33793451.73
2009	288000000	252293578	38600917.43
2010	313000000	275000000	44000000
2011	325000000	299777500	50062842.5
2012	338000000	327057252.5	56907961.94
2013	352000000	356525111	64531045.08
2014	367000000	388612370.9	73059125.74
2015	381000000	302000000	58890000
2016	350000000	242000000	48884000
2017	348000000	261000000	54549000
2018	359000000	269250000	57350250
2019	368000000	276000000	61548000
2020	367000000	275250000	63307500
2021	398195000	298646250	70779161.25
	9285195000	**6402590694**	**970248085**

In general, over the entire period of the data (1988-2021), approximately 80% of all plastics produced became waste; close to 14% of the total waste was recycled, about 21% incinerated and 64% discarded. The total rate of unrecovered was close to 85%. All the parameters are in line with major research that have uncovered the uncontrollable level of mismanaged plastic waste [12,13,14].

Now, the power of the model depends on the separation parameter β, which is defined by

$$\beta = w + (g - \psi), \tag{10}$$

where, g is the global annual plastic in-stock given by

$$g = \frac{\sum_{n=0}^{33}(x_n - y_n)}{\sum_{n=0}^{33} x_n}. \tag{11}$$

The next step is to compute the values of the other parameters $\beta, w, k, k_1, K, \lambda, K_1, K_2, K_3$ and K_4. These play significant roles in the validation process of the models. We compute the first five given below.

$$k = 0.081474229$$
$$k_1 = 1.081474229$$
$$K = 0.540737114$$
$$\lambda = 0.472071857$$

The value of λ is adjusted by $\lambda_\alpha = \frac{\lambda}{\mu} = \frac{0.472071857}{0.814742289} = 0.579412488$. This is to ensure that strength of the decay does not overpower that of the hyperbolic growth, else the values will decay continuously, and the data will not fit both $x(t)$ and (t) .

The predictive accuracy of the model largely depends on the remaining four: K_1, K_2, K_3 and K_4, which are functions of the initial values (x_0, y_0). It is important to stress that since the models are assumed to operate in a closed model, x_0 plays two alternating roles; either as the initial value for the volume of the global annual plastic products or the initial value of the global annual recycled plastics. The same ordered pair of initial values (x_0, y_0) cannot lead to higher predictive accuracy for both $x(t)$ and $y(t)$, so we employed a novel alternating technique we have labelled the "nth-degree product derivative proximity". In this approach, we compute the predicted values for a product by pairing its initial value with that of a product that is immediately derived from it. For example, if we consider $x(t)$, $y(t)$ is immediately derived from it so, $y(t)$ is a first-degree product derivative to $x(t)$. Similarly, where as x^{rec} is a first-degree product derivative to $y(t)$, it is a second-degree derivative to $x(t)$. Using the proposed method of *n^{th}-degree product derivative proximity,* we select the ordered pair $(x; y) = (95{,}000{,}000;\ 82{,}900{,}000)$ as initial values to compute the values of K_1 and K_2 to predict the values of $x(t)$. We obtain

$$K_1 = 226953163$$
$$K_2 = 118134928.9$$

Similarly, we select the ordered pair $(82{,}900{,}000; 497400)$ to compute the values of K_3 and K_4 to predict the values of $y(t)$. We obtain as differently, the following parameter values:

$$K_3 = 191055953.8$$
$$K_4 = 96097726.24$$

The predicted values of $x(t)$ at $\beta = w + (g - \psi) = 0.853580204 + (0.18525771 - 0.146419796)$

$$\beta = 0.892418119 \approx 0.9$$

can be located in column 4 of Table 2, in columns 3 and 5 are the predicted values of $x(t)$ at $\beta = 0.89$ and $\beta = 0.91$.

3 Results and Discussions

3.1 Predicting with the model

Applying Equation 5, we generate the predicted values of the global annual plastic production (Table 2).

Table 2. Historical values of the global annual plastic production against the corresponding predicted values (metric tonnes (Mt)) at different values of β

Year	Annual Plastic Production (Mt)	Predicted Annual Plastic Production at $\beta = 0.89$ (Mt)	Predicted Annual Plastic Production at $\beta = 0.9$ (Mt)	Annual Plastic Production at $\beta = 0.91$ (Mt) (Mt)
1988	95000000	95000000	95000000	95000000
1989	100000000	106132993.9	106996074.1	107866692.8
1990	105000000	112152012.3	113906636.2	115691095.3
1991	109000000	116579969.1	119276330.3	122040184.3
1992	115000000	120567676.5	124266189.2	128086545.8
1993	120000000	124492768.2	129259822.1	134221437
1994	130000000	128480917	134387409.3	140581428.2
1995	134000000	132575743.6	139696653.2	147220357.5
1996	145000000	136794205.7	145208555.9	154165488.9
1997	157000000	141144656.4	150935623.6	161435867.9
1998	165000000	145632733.9	156887813.6	169048334.8
1999	175000000	150263284	163074483.7	177019510.8
2000	185000000	155040989.9	169505036.9	185366470.5
2001	195000000	159970580.4	176189141.3	194106985.3
2002	204000000	165056900.9	183136811.9	203259629.3
2003	210000000	170304940.1	190358447.5	212843841.3
2004	225000000	175719841.6	197864852.9	222879972.4
2005	227000000	181306911.4	205667258.2	233389332.4
2006	240000000	193019627	222207217.7	255918047.8
2007	257000000	199156748.9	230969515.3	267985237.3
2008	245000000	205489002.6	240077336.7	280621425.6
2009	250000000	212022592.4	249544306.8	293853442.5
2010	270000000	218763919.8	259384587.9	307709382.7
2011	279000000	225719589.9	269612900.8	322218666
2012	288000000	232896417.9	280244546.9	337412098.9
2013	299000000	240301435.3	291295430.6	353321941.1
2014	311000000	247941897.8	302782083.9	369981972.9
2015	322000000	255825291.2	314721690.4	387427567.8
2016	335000000	263959339.7	327132111.4	405695767
2017	348000000	272352012.9	340031912.6	424825358.5
2018	359000000	281011533.9	353440391.6	444856959
2019	368000000	289946387.2	367377607.3	465833100.7
2020	367000000	299165327	381864409.1	487798321
2021	398195000	308677386.1	396922469.1	510799257.6
Total	**7732195000**	**6463465635**	**7529225658**	**8820481721**

The predicted values of the global annual plastic waste generation $y(t)$ at the different values of β are summarized in Table 3.

Table 3. Historical values of the global annual plastic waste generation against the corresponding predicted values (metric tonnes (Mt)) at different values of β

Year	Annual Plastic Wastes Generation (Mt)	Predicted Annual Plastic Wastes Generation at $\beta = 0.89$ (Mt)	Predicted Annual Plastic Wastes Generation at $\beta = 0.9$ (Mt)	Predicted Annual Plastic Wastes Generation at $\beta = 0.91$ (Mt)
1988	82900000	82900000	82900000	82900000
1989	86800000	90644712.68	90875318.34	91107602.71
1990	89500000	95192179.07	95994153.16	96806963.23
1991	93400000	98761727.2	100280369	101829835.4
1992	97400000	102078856.6	104398126.9	106780955.5
1993	102600000	105382175	108568165.2	111864785.5
1994	107900000	108751640.7	112866724.2	117155439.3
1995	113200000	112215564.6	117323071.2	122684822.6
1996	118400000	115785493.3	121951322.9	128471424.6
1997	126300000	119467582.2	126760833.5	134529735.2
1998	134200000	123266305.2	131759590.6	140873336.9
1999	142100000	127185666.8	136955331.1	147515933.6
2000	150000000	131229599.1	142355911.9	154471705.9
2001	160500000	135402094.3	147969439.7	161755447.6
2002	165800000	139707250.4	153804321.1	169382632.1
2003	171100000	144149288.7	159869287.9	177369456.5
2004	181600000	148732562.6	166173414.1	185732879.5
2005	192100000	153461562.9	172726130.7	194490659.2
2006	200000000	163375424.6	186616932.9	213264545.2
2007	207900000	168569999.5	193975798.8	223320513.1
2008	221100000	173929737.5	201624846.8	233850645.6
2009	223700000	179459890.2	209575519.8	244877300.7
2010	218400000	185165875.9	217839711.7	256423890.8
2011	227600000	191053285.3	226429785.5	268514932.1
2012	244700000	197127886.7	235358591.6	281176096.9
2013	252600000	203395632.1	244639487.4	294434267.9
2014	265800000	209862662.5	254286356.8	308317595.6
2015	300000000	216535314.2	264313631.2	322855557.6
2016	242000000	223420125.1	274736311.1	338079021.6
2017	261000000	230523840.8	285569988.6	354020310.9
2018	269250000	237853421.4	296830870.5	370713272.6
2019	276000000	245416048.5	308535802.8	388193350.1
2020	275250000	253219131.7	320702295.7	406497657.9
2021	298646250	261270316.5	333348549.9	425665060.6
Total	**6299746250**	**5474492854**	**6327915993**	**7355927635**

3.2 Validation of the model: The mean absolute percentage and the root mean square errors

The mean absolute percentage error (MAPE) provides a measure for testing the predictive accuracy of the model. It has been described in [15] as measure of a model's predictive power. It measures the average of the percentage absolute deviations of a model's predicted values from its observed or actual values. It provides an average measure of the margin below which the predicted values fall and above which the predicted values fall. Instances of MAPE application are easy to find [16,17,18]. By adopting the variables used in [1], the MAPE is given by

$$MAPE = \frac{1}{t}\sum_{t=0}^{34}\left|\frac{h_t - \bar{h_t}}{h_t} \times 100\right|, \quad (12)$$

where, h_t denotes the historical values while $\bar{h_t}$ is denotes predicted values.

Mathematically, the root mean square error (RMSE), is the square root of the mean of the squared deviations of the predicted values from the historical values. This can be defined by

$$RMSE = \sqrt{\frac{\sum_{t=0}^{33}\left(h_t - \bar{h_t}\right)^2}{t}}. \tag{13}$$

A close look at Equation 13 show the square root of the variance between the observed and the predicted values of the global annual plastic: production and waste generation. Therefore, the RMSE can be interpreted as the standard deviation of the unexplained variation between the historical and the predicted values. It is a good estimator for the standard deviation of the distribution of the errors [19]. An example of RMSE as a metric for evaluating predictive accuracy can be found in [20]. The smaller the RMSE, the better the fit, however, there is no standard measure of how small or big the RMSE should be. The application of the size of the RMSE in judging how better a model may fit a data depends on the values, range and units of the data. However, when two different models are tested on a data, the one that produces the smaller RMSE reflects a better fit. So, in the validation process of a model, picking the test value that produces the smallest RMSE is just enough to obtain a good fit. In this study, the predictions were done at different values of the separation target, then Equation 13 was applied to compute the RMSE corresponding to each value of the separation target. Subsequently, Equation 12 was applied to as a confirmation and to give a better interpretation of the both the explained and unexplained variations. From the predictions in Table 2, the RMSE corresponding to the following values of β; 0.89, 0.9 and 0.91 are respectively 52, 043, 566; 16, 660, 445; and 36, 347, 663 approximately. Similarly, we have RMSEs of 35, 842, 866; 16, 972, 407; and 41, 234, 768 respectively for the same respective values of β (as can be computed from Table 3). Therefore, the least value of RMSE computed is approximately 16, 660, 845 for the global annual plastic production and 16, 972, 407 for the global annual plastic waste generation which were obtained at $\beta = 0.9$. These values may seem too big, but this is normal considering the range of values involved in the global plastic data in Table 1. We can express this as a percentage of the means of the observed values of the global annual plastic: production and waste generation. This is called the scattered index (SI) which can be expressed as

$$SI = \frac{RMSE}{mean} \times 100$$

Thus, denoting by SI_x and SI_y, the SI respectively for the global annual plastic: production and waste generation, we have

$$SI_x = \frac{16,660,845}{227417500} \times 100 = 7.33\%$$

$$SI_y = \frac{16,972,407}{185286654.4} \times 100 = 9.16\%$$

Judging by the values of SI, we can be sure that both RMSEs are small in respect of the global plastic data. We now confirm from the MAPE that the least RMSE is associated with the least value of MAPE. Table 4 summarizes the computation of MAPE for both the global annual plastic production and the global annual plastic waste generation at the various values of β.

For the global annual plastic production $x(t)$, the MAPE corresponding to $\beta = 0.9$ is approximately 6.5%, indicating that on average, each predicted value of $x(t)$ deviates from its corresponding observed value by 6.5%. This implies that $x(t)$ can predict the observed values with a 93. 5% degree of accuracy. At $\beta = 0.89$, the MAPE is 16% approximately, which also implies that $x(t)$ can predict, on average, its corresponding observed values with an accuracy degree of approximately 84%. Finally, to the right of 0.9 ($\beta = 0.91$), the MAPE is approximately 9%; meaning that the observed values of global annual plastic production can be predicted with an accuracy degree of 91%. Further computations have revealed that any movement to the left or right of $\beta = 0.9$ will worsen the predictive accuracy of $x(t)$. Therefore, $x(t)$ predicts with the highest degree of accuracy at $\beta = 0.9$. It is worthy to note that at this value, the SI is equivalent to that of the MAPE by approximation.

Table 4. Computation of MAPE at $\beta = 0.89, 0.9$ and 0.91 for both global annual plastic production and waste generation

YEAR	MAPES FOR ANNUAL PRODUCTION $\left\lvert\frac{x_n - \overline{x_n}}{x_n} \times 100\right\rvert$			MAPES FOR ANNUAL WASTE GENERATION $\left\lvert\frac{y_n - \overline{y_n}}{y_n} \times 100\right\rvert$		
	$\beta = 0.89$	$\beta = 0.90$	$\beta = 0.91$	$\beta = 0.89$	$\beta = 0.90$	$\beta = 0.91$
1988	0	0	0	0	0	0
1989	6.13299388	6.99607413	7.86669283	4.429392	4.695067	4.962676
1990	6.8114403	8.48251071	10.1819956	6.359977	7.256037	8.164205
1991	6.95410006	9.42782593	11.9634718	5.740607	7.366562	9.02552
1992	4.84145781	8.05755583	11.3796051	4.803754	7.184935	9.631371
1993	3.7439735	7.71651843	11.8511975	2.711672	5.816925	9.030005
1994	1.16852535	3.3749302	8.13956018	0.789287	4.603081	8.577794
1995	1.06287788	4.2512337	9.86593841	0.869643	3.642289	8.378819
1996	5.6591685	0.14383166	6.32102686	2.208198	2.999428	8.506271
1997	10.098945	3.86266015	2.82539358	5.409674	0.364872	6.516022
1998	11.737737	4.91647661	2.45353622	8.147314	1.818487	4.97268
1999	14.1352663	6.81458075	1.15400619	10.49566	3.620457	3.811354
2000	16.1940595	8.37565575	0.19809217	12.5136	5.096059	2.981137
2001	17.9638049	9.64659421	0.45795627	15.63732	7.807203	0.78221
2002	19.0897544	10.227053	0.36292679	15.73748	7.23503	2.160816
2003	18.9024095	9.35312025	1.35421012	15.75144	6.563829	3.664206
2004	21.9022926	12.0600654	0.94223451	18.09881	8.494816	2.275815
2005	20.1291139	9.39768363	2.81468387	20.11371	10.0853	1.244487
2006	22.0534901	10.9261102	1.83093144	20.82954	10.23138	1.830695
2007	24.8950868	13.5380476	0.42099307	21.41634	10.23717	2.580349
2008	18.7115311	5.72672843	9.38172952	23.75848	12.26784	1.004303
2019	17.8043989	3.96906533	12.2485703	22.24866	9.868195	4.537615
2010	21.4731139	7.57618268	8.83460833	17.82972	4.040513	12.12331
2011	21.5899929	7.03061366	10.2901013	18.64417	4.288352	12.66428
2012	21.6251424	6.38440944	11.8814812	21.92346	7.466373	9.732298
2013	22.1082215	6.2727268	12.8468558	21.96046	6.825577	11.31279
2014	22.7326574	6.3358744	13.6083412	23.47794	7.961066	10.77286
2015	22.9994106	5.96829693	14.9012338	30.04578	15.23788	2.772532
2016	23.6342414	6.05322675	15.6500202	10.5226	9.220509	33.41139
2017	24.149615	5.99651972	16.5792434	14.39842	5.262954	29.53219
2018	24.1359295	5.28358982	18.3357544	14.38297	6.061277	31.48387
2019	23.6381701	3.95641532	20.8850432	13.82122	7.547417	34.3164
2020	20.9955348	0.10289026	26.9300002	10.83886	12.09293	41.03301
2021	24.8696425	4.10115417	22.5023722	15.21101	7.385342	36.11343
Total	**543.9441**	**222.326222**	**307.259808**	**451.1272**	**230.6452**	**369.9067**
MAPE	**15.9983559**	**6.53900652**	**9.03705317**	**13.26845**	**6.783681**	**10.87961**

Bringing into perspective the global annual plastic waste generation $y(t)$, we have MAPE $= 7\%$ approximately for $\beta = 0.9$, implying that on average, each predicted value of $x(t)$ varies from its corresponding observed value by 7%. This means that $x(t)$ can predict the observed values with an approximately 93% degree of accuracy. Also, at $\beta = 0.89$, the MAPE is 13.3% approximately, which implies that $y(t)$ predicts the corresponding observed values with an accuracy degree of about 86.7%, on average. Further, at $\beta = 0.91$, the

MAPE is approximately 11%; indicating that $y(t)$ predict the corresponding observed values of global annual plastic waste generation with an approximately 89% degree of accuracy. Further analyses have uncovered that any variation to the left or right of $\beta = 0.9$ will worsen the predictive accuracy of $y(t)$. Thus, we conclude that $y(t)$ predicts with the highest degree of accuracy at $\beta = 0.9$. It is imperative to compare the closeness of the values of the MAPEs and SIs.

In summary, at a separation target of $\beta = 0.9$, Equation 5 predicted the historical values of the global annual plastic production with a MAPE of 6.5% approximately, indicating that on average, the predicted values of the global annual plastic production will fall below or above the corresponding historical values by 6.5%. It can therefore be established that Equation 5 predicts the values of the global annual plastic production with 93.5% degree of accuracy. Similarly, with a MAPE value of approximately 7%, the Equation 6 predicts the values of the global annual plastic waste generation, implying that Equation 6 can predict the historical values of global annual plastic waste generation with approximately 93% degree of accuracy. In [1], the MAPEs for the non-separation-based CDC model were approximately 13% for the global annual plastic production model and 18% for the global annual plastic waste generation model. Hence, non-separation-based CDC model predicted with approximately, 87% and 82% degrees of accuracies respectively for the global annual plastic: production and waste generation. From the perspective of the RMSE, the existing CDC model was associated with RMSEs of 33, 708, 488 approximately for the global annual plastic production and 30, 815, 434 for the global annual plastic waste generation. Clearly, the separation-based CDC model in this current study has outperformed the non-separation-based CDC model developed by Addor et al. (2022); thereby establishing its predictive supremacy.

Two significant elements determined the strength of the CDC model in this current study; the separation target, which is a very crucial determinant of both quality and quantity of recycled plastics; and the nth-order product derivative proximity technique applied in the selection of initial values. The computational processes revealed that there would have been worse performance if these two elements were not used together.

Based on the MAPE analyses, a visual summary of the historical against the predicted values of the global annual: plastic production and waste generation are presented at different values of β (Figs. 1 and 2).

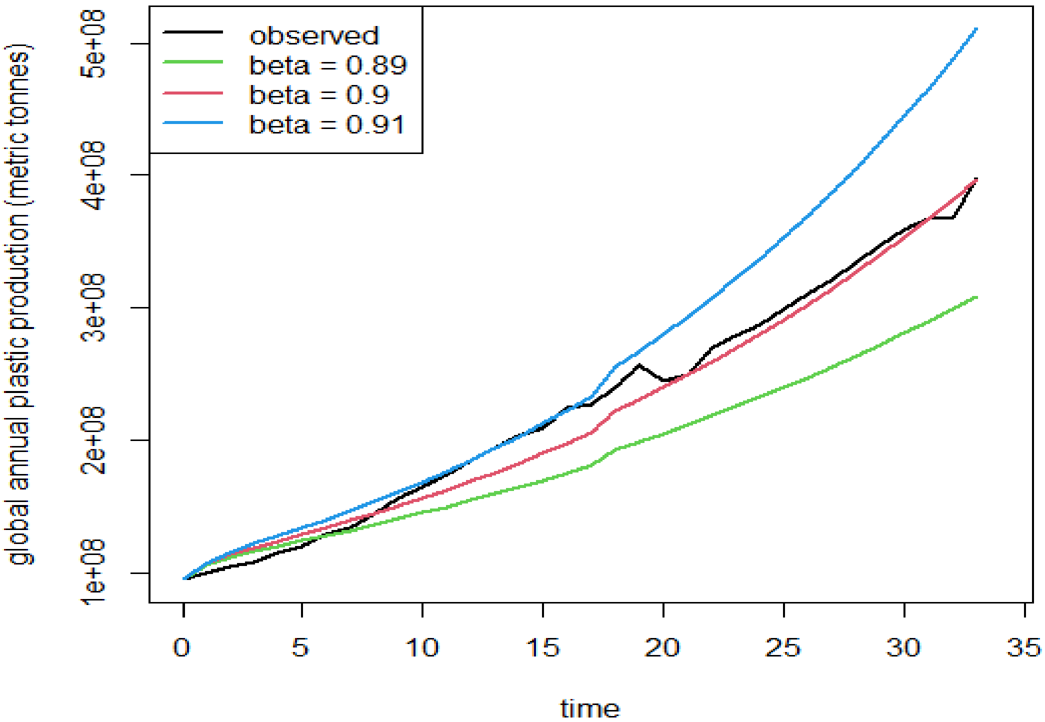

Fig. 1. A time series plot of the historical against predicted values of $x(t)$ at $\beta = 0.89, 0.9$ and 0.91

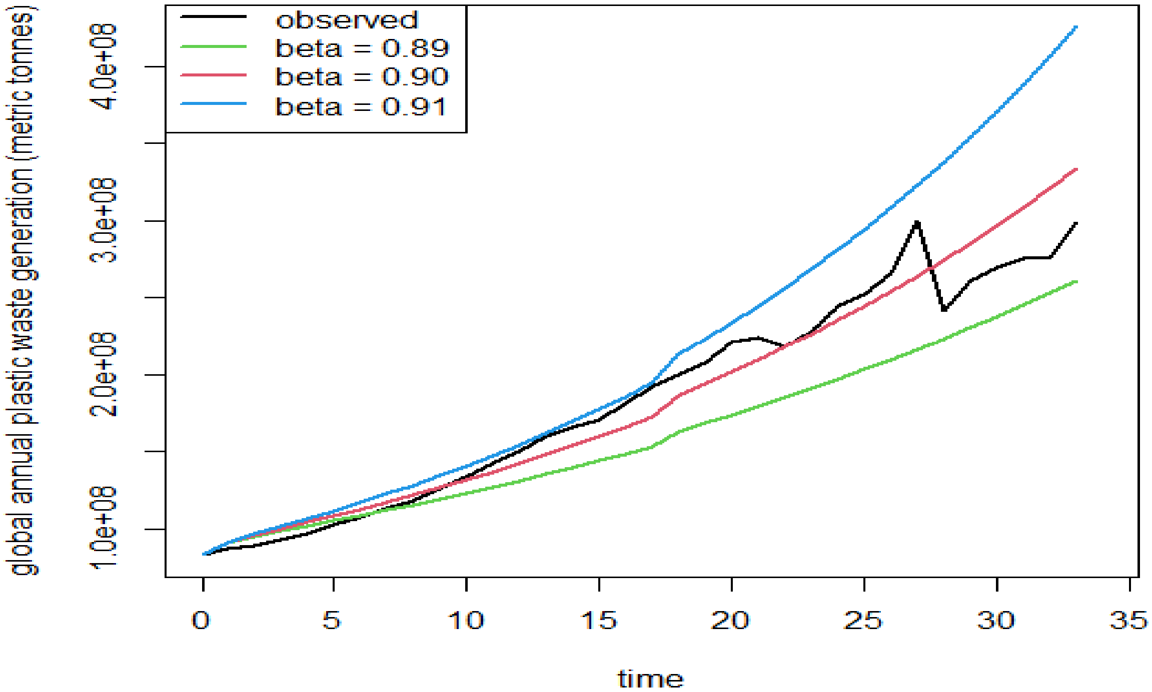

Fig. 2. A time series plot of the historical against the predicted values of $y(t)$ at $\beta = 0.89, 0.90$ and 0.91

3.3 Forecasting with the model

In this section, we apply the best predicting models to forecast for a 29-year period from 2022 through to 2050. The purpose is to compare the cumulative values of global annual plastic production and waste generation with existing findings, especially, the existing CDC model. The results of the forecast are depicted in Figs. 3 and 4.

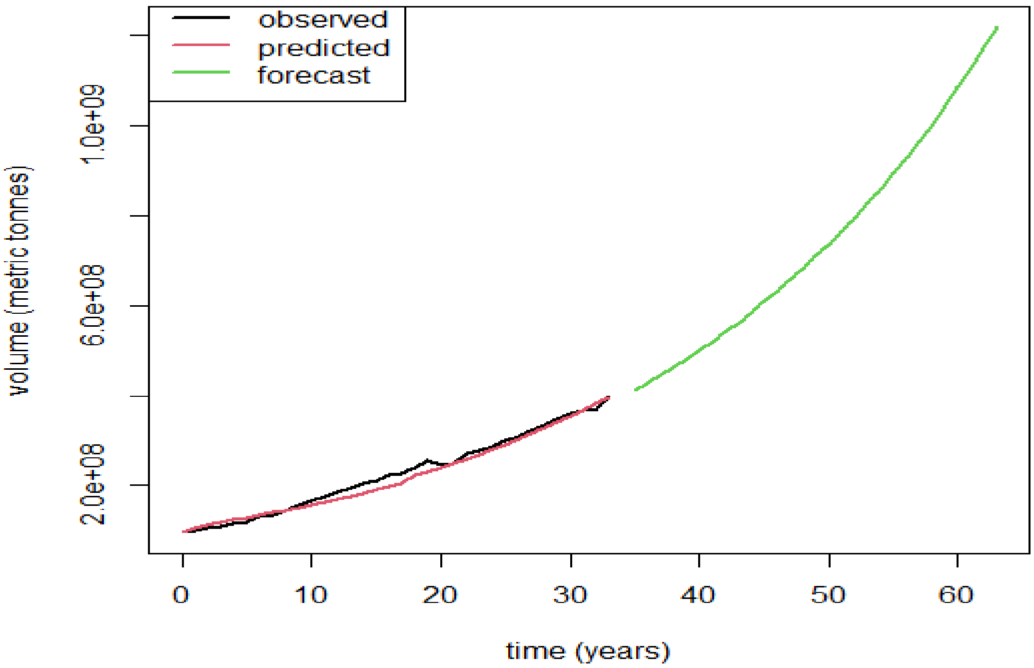

Fig. 3. A plot of a 29-year (2022-2050) forecast for the volume of global annual plastic production

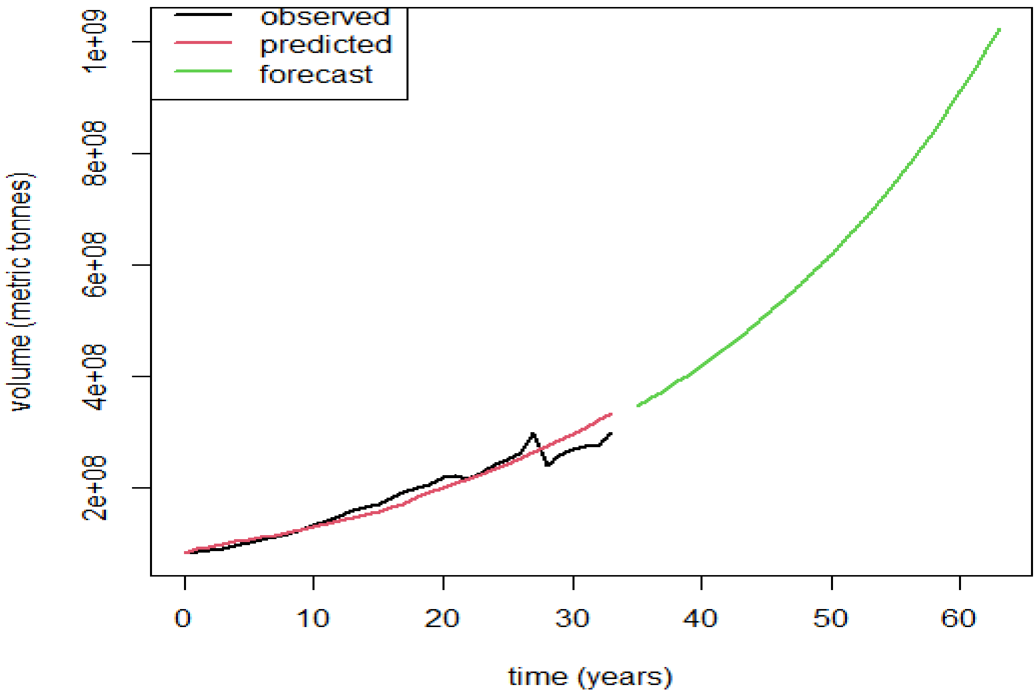

Fig. 4. A plot of a 29-year (2022-2050) forecast for the volume of global annual plastic waste generation

The predictions by the models revealed that from 1988 through to 2021, cumulative volumes of approximately 7530 (million metric tonnes) or 7.53 (billion metric tonnes); and 6328 (million metric tonnes) or 6.33 (billion metric tonnes) were estimated for the global annual plastic production and global annual plastic waste generation respectively. This shows that global cumulative plastic production and waste generation for 33 (1988 – 2021) years is about the same as that produced over a period of 65 (1950 – 2015) years as estimated in [2]. This means that plastic production has increased more than two times over the past 33 years.

The model was used to forecast the volumes of global annual plastic: production and waste generation over a period of 29 years (2022-2050). The forecast estimated a global annual plastic production of 1219 (million metric tonnes) or (1.22 billion metric tonnes) and a global annual waste generation of 1023 (million metric tonnes) or (1.02 billion metric tonnes) by the year 2050. Finally, cumulative volumes of global annual plastic: production and waste generation from 1988 to 2050 are 29.2 (billion metric tonnes) and 24.52 (billion metric tonnes) approximately. This implies that an annual average of 1007 (million metric tonnes) and 846 (million metric tonnes) respectively for global plastic: production and waste generation.

In [21], the cumulative global plastic production was projected to about 34 (billion metric tonnes), and plastic waste generation to about 12 (billion metric tonnes) by the year 2050. Cumulative global plastic waste generation was estimated to 12 (billion metric tonnes) by 2050 [2]. Additionally, a-2050 projection of 1600 million metric tonnes was made for global annual plastic production in [22]. Last but not least, in [1], it was estimated that by 2050, a cumulative value of approximately 43.2 (billion metric tonnes) of plastic will be produced; and approximately 17.8 (billion metric tonnes) of plastic wastes will be generated. By adjusting our production values with plastic production data from 1950 to 1987 [2] our total cumulative estimate for global annual plastic production by the year 2050 is approximately 31 (billion metric tonnes). We can then assess the closeness of the 2050 estimated cumulative global annual plastic production by Statista (2022) to our forecast value.

A comparison with our 29 years (2022-2050) forecast makes clear the difference, which can be explained by the difference in computational techniques as well as the base years applied. In the referenced studies, the base year was 1950. Another explanation can also be ascribed to the fact that the estimates made by [2] to 2050 was based on a projection of an assumed constant growth rate of 0.07%. However, according to [23,24], increase in plastic production and waste can be explained by rapid population growth, which follows exponential growth path.

Lastly, another reason may arise out of the different polymer types that characterize each plastic data. That notwithstanding, some techniques are more superior to others, which assertion has been established by the MAPEs of the improved CDC model.

4 Conclusion

The new CDC model for plastic waste management has outperformed the existing CDC model. The model's outperformance links to the inclusion of the separation target and the nth-order product derivative proximity technique applied in the selection and paring of initial values. The RMSE associated with the global annual plastic production of the new CDC model is smaller (RMSE = 16, 660, 845) than that of the existing CDC model (RMSE = 33, 708, 488). Similarly, the RMSE associated with the global annual plastic waste generation in the new CDC model is smaller (RMSE = 16, 972, 407) relative to that of the existing CDC model (RMSE = 30, 815, 434). Furthermore, the existing CDC model had a MAPE of 13% approximately for the global annual plastic production and 18% for the global annual plastic waste generation. In comparison, the MAPEs associated with the improved CDC model are 6.5% for the global annual plastic production and 7% approximately for the global annual plastic waste generation. Therefore, the improved CDC model predicts with 93.5% and 93% degrees of accuracy respectively for the global annual plastic production and the global annual plastic waste generation. The predicted cumulative estimates from 1988 through to 2021 are 7530 (billion metric tonnes) for the global annual plastic production and 6328 (billion metric tonnes) for the global annual plastic waste generation. The 29 years (2022 – 2050) forecast revealed a yearly estimate of 1.22 (billion metric tonnes) and 1.02 (billion metric tonnes) respectively for the global annual plastic production and the global annual plastic waste generation. By 2050, cumulative estimates for the global annual plastic production and the global annual plastic waste generation are respectively 29.2 (billion metric tonnes) and 24.52 (million metric tonnes). Thus, averagely, the global annual plastic production and global annual plastic waste generation are estimated to be 1007 (million metric tonnes) and 846 (million metric tonnes), respectively. The model can therefore make impactful policy decisions for sustainable plastic waste management thereby aiding to achieve the transition towards circular economy in plastic waste management.

Disclaimer

The products used for this research are commonly and predominantly used products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

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Competing Interests

Authors have declared that no competing interests exist.

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