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# Linear Estimation in the Type II Generalized Logistic Distribution under Progressive Censoring

**Rana Rimawi [a] and Ayman Baklizi [a*]**

*[a] Statistics Program, Department of Mathematics, Statistics and Physics, College of Arts and Science, Qatar University, 2713, Doha, Qatar.*

***Authors' contributions***

*This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.*

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## Abstract

Generalized distributions have become increasingly popular in applications. They are highly flexible in data analysis, especially with skewed data, which are common in many applications. The Generalized Logistic Distribution (GLD) and its special cases have recently received a lot of interest in the literature. We derived estimators of the unknown parameters of type II Generalized Logistic Distribution (Type II GLD) based on progressively type II censored data. A variety of point estimation methods is employed. We considered the best linear unbiased estimator (BLUE) and the best (affine) linear equivariant estimator (BLEE). In addition, we considered Bayesian estimation. Simulation approaches were used to study the estimators and compare them with the maximum likelihood estimator (MLE) in a range of progressive censoring schemes. The mean squared error (MSE) and bias were employed as comparison criteria. An example based on real data is presented.

*Keywords: Point estimation; best linear unbiased estimation; best linear equivariant estimation; type II generalized logistic distribution, progressive censoring.*

## 1 Introduction

Considerable attention has been paid in the literature to inference in parametric distributions based on progressively censored data. Balakrishnan and Sandhu [1] considered progressive Type II censored sample to

**Corresponding author: Email: a.baklizi@qu.edu.qa;*

find the best linear unbiased estimators to estimate the parameters of the exponential distributions. In addition, they found the maximum likelihood estimators (MLE's) and found that they are equal to the BLUE's of the two-parameter exponential distribution. Also, they drew the attention to the fact that the accuracy of the estimators of the location and scale parameters (BLUE) depends on r, n and m but not the progressive censoring scheme R. The generalized exponential distribution was studied by Kundu and Pradhan (2009). They considered Bayesian inference of the parameters of based on the progressively censored data assuming independent gamma priors for the scale and shape parameters. Bayes estimates are approximated using Lindley's approximation as well as importance sampling using Markov chain Monte Carlo techniques. The authors noted that the Bayes estimates have strong advantages over the MLEs, if suitable prior information is available. The generalized Rayleigh distribution was considered by Maiti and Kayal (2019) where they considered estimation of parameters and reliability characteristics a under progressive type-II censored sample. The MLEs and Bayes estimates of the parameters were obtained under various loss functions. Salah [2] considered estimating the unknown parameters of $\alpha$-power exponential distribution under progressively Type II censored data using the MLEs. He found the approximate best linear unbiased estimators (ABLUE's) as an initial guess of the MLEs. The author discovered that ABLUEs and MLEs are closely related in the case of the exponential distribution with two parameters. This closeness provides good initial estimates of MLEs. Aly and Bleed (2013) considered Bayesian estimation of the generalized logistic distribution based on progressively censored data under accelerated testing.

In this paper, we shall consider the type II generalized logistic distribution whose probability density function is given by:

$$f(x|\lambda,\mu,\sigma) = \frac{\lambda^{\alpha}}{\sigma\Gamma(\alpha)}\exp[-\alpha\frac{x-\mu}{\sigma}]\exp[-\lambda\exp\frac{x-\mu}{\sigma}], -\infty < x,\mu < \infty; \sigma,\alpha,\lambda > 0. \quad (1)$$

Nassar and Elmasri [3]; Azizpour and Asgharzadeh [4] and Aljarrah et al. [5] studied MLEs for the Generalized Logistic Distribution and other distributions under progressive censoring. Balakrishnan and Hossain [6] found that the approximate maximum likelihood estimators (AMLEs) and the MLEs have similar performance in terms of bias and variance. Moreover, Rimawi and Baklizi [7] investigated the type II Generalized Logistic Distribution estimators based on type II progressive censoring data. They analyzed the MLE and the Lindley's approximation to the Bayes estimator.

In this work, we will derive approximate linear estimators of the parameters of the type II generalized logistic distribution using type II progressively censored data. Progressive censoring is a type of censoring where we have n units that are placed simultaneously on the life-testing experiment. Immediately following the first failure, $r_1$ surviving units are randomly chosen and removed from the experiment. Immediately after the second failure, $r_2$ items are withdrawn and so on. The procedure is continued until all $r_m$ remaining units are removed after the $m^{th}$ failure. Note that the $r_i$'s are fixed prior to study. If $r_1 = r_2 = \ldots = r_m = 0$, then n = m which corresponds to the complete sample, while when $r_1 = r_2 = \ldots = r_{m-1} = 0$, we have $r_m = n - m$ which corresponds to the conventional Type II right-censoring scheme.

# 2 Approximate Best Linear Unbiased Estimators

Linear statistics have an easy and accurate structure. Researchers have been interested in using linear inference for parametric distributions with ordered data in a variety of applications because of their ease and accuracy. Suppose we have $(\boldsymbol{X} = X_{1:m:n}, \ldots, X_{m:m:n})$` be a random vector of progressively Type-II censored order statistics from a distribution with location parameter μ and scale parameter σ. Let Y = $(Y_{1:m:n}, \ldots, Y_{m:m:n})$` be such that:

$$Y_{j:m:n} = \frac{X_{j:m:n} - \mu}{\sigma}, j = 1, \ldots, m. \quad (2)$$

Let $W = \sigma(Y - E(Y))$, $b = E(Y)$, $\theta = (\mu, \sigma)$` *and* $B = [\mathbb{1}, b]$. It follows that $X$ can be presented as a linear equation:

$$X = \mu.\mathbb{1} + \sigma.Y = \mu.\mathbb{1} + \sigma.E(Y) + W = [\mathbb{1}, b]\binom{\mu}{\sigma} + W = B\,\theta + W`. \quad (3)$$

Let Σ be the covariance matrix cov(Y), assuming Σ is regular, and non-singular covariance matrix, then:

$$\Sigma = \Delta \Sigma_{U} \Delta. \quad (4)$$

The best linear unbiased estimator (BLUE) for the parameters under study depends on the evaluation of the variance covariance matrix of the order statistics from the progressively censored data. This matrix is very complicated and can not be obtained in closed form. An approximate best linear unbiased estimator is available. It is derived in Balakrishnan and Cramer [8]. We will apply this approximation to the location and scale parameters of our model as follows:

Suppose we have $m \geq 2$ and $n = \sum_{j=1}^{m} r_j + 1$ , the BLUE estimators of μ and $\sigma$ are given by:

$$\hat{\mu}_{LU} = \frac{1}{\Delta}.\left((b`\Sigma^{-1} b)(\mathbb{I}` \Sigma^{-1}X) - (\mathbb{I}` \Sigma^{-1}b)(b`\Sigma^{-1} X)\right), \quad (5)$$

$$\hat{\sigma}_{LU} = \frac{1}{\Delta}.\left((\mathbb{I}`\Sigma^{-1} \mathbb{I})(b` \Sigma^{-1}X) - (\mathbb{I}` \Sigma^{-1}b)(\mathbb{I}`\Sigma^{-1} X)\right), \quad (6)$$

where $\Delta = \left((\mathbb{I}`\Sigma^{-1} \mathbb{I})(b` \Sigma^{-1}b) - (\mathbb{I}`\Sigma^{-1} b\right)^2 > 0$.

In order to find the approximate covariance matrix, we calculate the following quantities;

$$\gamma_j = n - j + 1,\ j = 1, \dots, n \quad ,\ c_r = \prod_{j=1}^{r} \gamma_j,\ r = 1, \dots, m,\ d_r = \prod_{j=1}^{r} (\gamma_j + 1), r = 1, \dots, m,$$
$$e_r = \prod_{j=1}^{r} (\gamma_j + 2),\ r = 1, \dots, m,\ a_r = \frac{d_r}{e_r},\ r = 1, \dots, m,\ b_r = \frac{c_r}{d_r}, r = 1, \dots, m,$$
$$EU_r = \Pi_r = 1 - b_r,\ r = 1, \dots, m,\ COVU_rU_s = (a_r - b_r)b_s, r = 1, \dots, m, s = 1, \dots, m.$$

The last quantity $COVU_rU_s$ gives the approximate covariance matrix $\Sigma_{U}$. Now Calculate the diagonal matrix Δ with diagonal elements $\left(\frac{1}{f(F^{-1}(\Pi_1))}, \dots, \frac{1}{f(F^{-1}(\Pi_r))}\right)$ where:

$f(x) = \frac{e^{-\alpha\left(\frac{x_i-\mu}{\sigma}\right)}}{\left(1+e^{-\left(\frac{x_i-\mu}{\sigma}\right)}\right)^{\alpha+1}}$ and $F(x) = 1 - \left[\left(\frac{e^{-\left(\frac{x_i-\mu}{\sigma}\right)}}{1+e^{-\left(\frac{x_i-\mu}{\sigma}\right)}}\right)^{\alpha}\right]$. We obtain the required covariance matrix, $\Sigma = \Delta \Sigma_{U} \Delta$.

The best linear equivariant estimators (BLEE) are approximated in a similar manner. Using the same notation used for the BLUEs, and let $\Delta_1 = \Delta + \left((\mathbb{I}`\Sigma^{-1} \mathbb{I}\right)$ we obtain:

$$\hat{\mu}_{LE} = \frac{1}{\Delta_1}.\left((b`\Sigma^{-1} b + 1)(\mathbb{I}` \Sigma^{-1}X) - (\mathbb{I}` \Sigma^{-1}b)(b`\Sigma^{-1} X)\right), \quad (7)$$

put sigma-hat-LE here, similar to equation 6.

$$= \frac{1}{\Delta_1}.\left((\mathbb{I}`\Sigma^{-1} \mathbb{I})(b` \Sigma^{-1}X) - (\mathbb{I}` \Sigma^{-1}b)(\mathbb{I}`\Sigma^{-1} X)\right). \quad (8)$$

# 3 Bayesian Estimation of Location and Scale Parameters

Bayesian statistical methods begin with established 'prior' beliefs and update them with data to generate 'posterior' beliefs that can be used to make inferences. Based on this technique, we will derive Bayes estimators for the parameters of the type II generalized logistic distribution (GLD) location and scale parameters (μ and σ) with type II progressively censored data.

To facilitate comparison with the classical estimators, we will assume non-informative prior distributions for the location and scale parameters, that is, $\pi(\mu) = 1$ and $\pi(\sigma) = 1/\sigma$. The likelihood function is given by:

$$l(data|\alpha, \mu, \sigma) \quad \alpha \quad \frac{1}{\sigma^m} \prod_{i=1}^{m} f(z_{i:m:n})[1 - F(z_{i:m:n})]^{r_i}. \quad (9)$$

Therefore, the joint posterior density of, $\mu$ and $\sigma$ given the data, is given by:

$$\pi(\mu, \sigma|data) \propto \frac{1}{\sigma} l(data|\mu, \sigma), -\infty < \mu < \infty, \sigma > 0. \quad (10)$$

The Bayes estimator of a function of the parameters, say $t = t(\mu , \sigma)$ under the squared error loss function is given by its posterior expectation:

$$\hat{t} = \int_0^\infty \int_{-\infty}^\infty \mathrm{t}(\mu , \sigma)\pi(\mu, \sigma|data) d\mathrm{\mu} d\mathrm{\sigma}. \qquad (11)$$

This integral is difficult to obtain analytically and therefore we can approximate it using either importance sampling procedures or the Lindley approximation.

Importance Sampling can be explained as a weighted average of random samples taken from another distribution $\square_v$(x) "importance sampling" density function to estimate an expectation with respect to the target density function $f_x$(x). The prior distribution of $\mu$ and $\sigma$ are non-informative priors for the location and scale parameters (μ and σ):

$$\pi_1(\mathrm{\mu}) = 1, -\infty < \mu < \infty, \qquad (12)$$

$$\pi_2(\mathrm{\sigma}) = \frac{1}{\sigma}, \mathrm{\sigma} > 0. \qquad (13)$$

The joint prior distribution is

$$\pi(\mu, \sigma) = \frac{1}{\sigma}, -\infty < \mu < \infty, \mathrm{\sigma} > 0. \qquad (14)$$

It follows that the posterior distribution is given by:

$$\pi(\mu, \sigma|data) = k \frac{\alpha^m}{\sigma^{m+1}} \prod_{i=1}^m \left\{ \frac{1}{\left(1+e^{-\left(\frac{x_i-\mu}{\sigma}\right)}\right)} \left( \frac{e^{-\left(\frac{x_i-\mu}{\sigma}\right)}}{1+e^{-\left(\frac{x_i-\mu}{\sigma}\right)}} \right)^{\alpha(r_i+1)} \right\}.$$

$$\propto \left\{ \frac{e^{m/\sigma}}{m^{m-1}} \left(1 + e^{-\left(\frac{\mu-\bar{x}}{\sigma/m}\right)}\right)^2 \prod_{i=1}^m \left\{ \frac{e^{-(\alpha(r_i+1)-1)\left(\frac{x_i-\mu}{\sigma}\right)}}{\left(1+e^{-\left(\frac{x_i-\mu}{\sigma}\right)}\right)^{\alpha(r_i+1)+1}} \right\} \right\}. \qquad (15)$$

We can rewrite the posterior function as:

$$\pi(\mu, \sigma|data) \propto \ f_1(\mu) f_2(\sigma) \square(\mu, \sigma), \qquad (16)$$

where $f_1(\mu) = \left\{ \frac{m}{\sigma} \frac{e^{\frac{\mu-\bar{x}}{\sigma/m}}}{\left(1+e^{\frac{\mu-\bar{x}}{\sigma/m}}\right)^2} \right\}$, this is the logistic distribution with parameters $\bar{x} = \frac{\sum_{i=1}^m x_i}{m}$ and $\sigma/m$. $f_2(\sigma) = \left\{ \frac{m^{m-1}}{\Gamma(m-1)} \left(\frac{1}{\sigma}\right)^m e^{-m/\sigma} \right\}$, which is the inverse gamma distribution's pdf with parameters $\mathrm{m} - 1$ and m, and

$$\square(\mu, \sigma) = \left\{ \frac{e^{m/\sigma}}{m^{m-1}} \left(1 + e^{-\left(\frac{\mu-\bar{x}}{\sigma/m}\right)}\right)^2 \prod_{i=1}^m \left\{ \frac{e^{-(\alpha(r_i+1)-1)\left(\frac{x_i-\mu}{\sigma}\right)}}{\left(1+e^{-\left(\frac{x_i-\mu}{\sigma}\right)}\right)^{\alpha(r_i+1)+1}} \right\} \right\} \qquad (17)$$

To find the estimate of the GLD parameters we do the following steps:

*<u>Algorithm</u> 1:*

Step 1: Generate $\sigma$ from inverse gamma distribution with parameters $m - 1$ and $m$.

Step 2: Generate $\mu$ from the logistic distribution with parameters $\bar{x} = \frac{\sum_{i=1}^m x_i}{m}$ and $\sigma/m$, where $\sigma$ is generated from Step 1.

Step 3: Repeat steps 1 and 2 to obtain $((\mathrm{\mu}_1, \mathrm{\sigma}_1), (\mathrm{\mu}_2, \mathrm{\sigma}_2), \ldots, (\mathrm{\mu}_N, \mathrm{\sigma}_N))$.

Step 4: Calculate the Bayes estimate as $\sum_{i=1}^N t(\mathrm{\mu}_i, \mathrm{\sigma}_i) \square((\mathrm{\mu}_i, \mathrm{\sigma}_i) / \sum_{i=1}^N \square((\mathrm{\mu}_i, \mathrm{\sigma}_i)$.

# 4 Simulation Study

A Monte Carlo simulation study is conducted to investigate and compare the performance of the estimators under various experimental situations. We considered various progressive censoring schemes as explained in Tables 1 – 6 below, corresponding to sample sizes of 50, 70 and 90. The location and scale parameters were set to zero and one respectively. The parameter $\alpha$ is taken to be 0.5, 1 and 1.5 to cover the various shapes of the distribution. We used the algorithm proposed by Balakrishnan and Sandhu [9] to generate progressive Type II censored samples from Type II GLD. The findings are presented in Tables 1 and 6. We used 5000 replications in all our simulation runs.

The results include the biases and mean squared errors for the estimators developed in this paper in addition to the Lindley's approximation of the Bayes estimators and the maximum likelihood estimators developed and studied in Balakrishnan and Hossain [6] and Rimawi and Baklizi [7].

**Table 1. Results of simulation for parameter μ with GLD (α =1.5, $\mu$ = 0, , $\sigma$ =1)**

| N | m | Scheme | MLE | Lindley | I.S | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| 50 | 30 | (0*29,20) | | | | | |
| | | Bias | -0.0316 | -0.0411 | -1.7436 | 0.0295 | 0.0101 |
| | | MSE | 0.0010 | 0.0017 | 3.0400 | 0.0660 | 0.0648 |
| | 30 | (0*10,2*10,0*10) | | | | | |
| | | Bias | -0.0293 | -0.0466 | -1.3551 | 2.2187 | 2.1775 |
| | | MSE | 0.0009 | 0.0022 | 1.8362 | 4.9878 | 0.0648 |
| | 30 | (20,0*29) | | | | | |
| | | Bias | -0.0092 | -0.0929 | -0.8390 | 2.6077 | 2.5681 |
| | | MSE | 0.0001 | 0.0086 | 0.7040 | 6.8653 | 0.0648 |
| 50 | 40 | (0*39,10) | | | | | |
| | | Bias | -0.0160 | -0.0226 | -1.2661 | 0.0172 | 0.0094 |
| | | MSE | 0.0003 | 0.0005 | 1.6030 | 0.0497 | 0.0493 |
| | 40 | (0*15,1*10,0*15) | | | | | |
| | | Bias | -0.0137 | -0.0421 | -1.0062 | 0.9233 | 0.9108 |
| | | MSE | 0.0002 | 0.0018 | 1.0125 | 0.9019 | 0.0493 |
| | 40 | (10,0*39) | | | | | |
| | | Bias | -0.0067 | -0.0586 | -0.7654 | 1.1288 | 1.1166 |
| | | MSE | 0.0000 | 0.0034 | 0.5858 | 1.3237 | 0.0493 |
| 70 | 40 | (0*39,30) | | | | | |
| | | Bias | -0.0246 | -0.0294 | -1.7559 | 0.0285 | 0.0129 |
| | | MSE | 0.0006 | 0.0009 | 3.0832 | 0.0506 | 0.0495 |
| | 40 | (0*10,2*15,0*15) | | | | | |
| | | Bias | -0.0246 | -0.0366 | -1.2942 | 2.6859 | 2.6498 |
| | | MSE | 0.0006 | 0.0013 | 1.6750 | 7.2640 | 0.0495 |
| 70 | 50 | (0*49,20) | | | | | |
| | | Bias | -0.0147 | -0.0224 | -1.4289 | 0.0164 | 0.0085 |
| | | MSE | 0.0002 | 0.0005 | 2.0419 | 0.0389 | 0.0385 |
| | 50 | (0*20,2*10,0*20) | | | | | |
| | | Bias | -0.0166 | -0.0557 | -1.0992 | 1.5217 | 1.5064 |
| | | MSE | 0.0003 | 0.0031 | 1.2083 | 2.3542 | 0.0385 |
| | 50 | (20,0*49) | | | | | |
| | | Bias | -0.0101 | -0.0557 | -0.7403 | 1.8189 | 1.8040 |
| | | MSE | 0.0001 | 0.0031 | 0.5481 | 3.3470 | 0.0385 |
| 90 | 50 | (0*49,40) | | | | | |
| | | Bias | -0.0248 | -0.0259 | -1.7668 | 0.0183 | 0.0053 |
| | | MSE | 0.0006 | 0.0007 | 3.1217 | 0.0406 | 0.0401 |
| | 50 | (0*15,2*20,0*15) | | | | | |
| | | Bias | -0.0153 | -0.0312 | -1.3673 | 2.8937 | 2.8620 |
| | | MSE | 0.0002 | 0.0010 | 1.8696 | 8.4135 | 0.0401 |

| N | m | Scheme | MLE | Lindley | I.S | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| 90 | 60 | (0*59,30) | | | | | |
| | | Bias | -0.0076 | -0.0180 | -1.5100 | 0.0143 | 0.0067 |
| | | MSE | 0.0001 | 0.0003 | 2.2800 | 0.0323 | 0.0321 |
| | 60 | (0*20,2*15,0*25) | | | | | |
| | | Bias | -0.0067 | -0.0252 | -1.1241 | 2.0089 | 1.9925 |
| | | MSE | 0.0000 | 0.0006 | 1.2636 | 4.0679 | 0.0321 |
| | 60 | (30,0*59) | | | | | |
| | | Bias | -0.0029 | -0.0420 | -0.7201 | 2.2792 | 2.2635 |
| | | MSE | 0.0000 | 0.0018 | 0.5185 | 5.2268 | 0.0321 |

**Table 2. Results of Simulation for parameter μ with GLD (α =1.0, $\mu$ = 0, , $\sigma$ =1)**

| N | m | Scheme | MLE | Lindley | I.S | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| 50 | 30 | (0*29,20) | | | | | |
| | | Bias | -0.0145 | -0.0260 | -1.2894 | 0.0078 | -0.0010 |
| | | MSE | 0.0002 | 0.0007 | 1.6625 | 0.0649 | 0.0648 |
| | 30 | (0*10,2*10,0*10) | | | | | |
| | | Bias | -0.0223 | -0.0400 | -0.8053 | 1.8900 | 1.8698 |
| | | MSE | 0.0005 | 0.0016 | 0.6485 | 3.6369 | 0.0648 |
| | 30 | (20,0*29) | | | | | |
| | | Bias | -0.0030 | -0.0845 | -0.2378 | 2.4078 | 2.3881 |
| | | MSE | 0.0000 | 0.0071 | 0.0565 | 5.8622 | 0.0648 |
| 50 | 40 | (0*39,10) | | | | | |
| | | Bias | -0.0044 | -0.0148 | -0.7395 | -0.0040 | -0.0056 |
| | | MSE | 0.0000 | 0.0002 | 0.5468 | 0.0584 | 0.0584 |
| | 40 | (0*15,1*10,0*15) | | | | | |
| | | Bias | -0.0108 | -0.0322 | -0.4200 | 0.6519 | 0.6492 |
| | | MSE | 0.0001 | 0.0010 | 0.1764 | 0.4834 | 0.0584 |
| | 40 | (10,0*39) | | | | | |
| | | Bias | 0.0044 | -0.0779 | -0.1488 | 0.9265 | 0.9239 |
| | | MSE | 0.0000 | 0.0061 | 0.0221 | 0.9169 | 0.0584 |
| 70 | 40 | (0*39,30) | | | | | |
| | | Bias | -0.0140 | -0.0206 | -1.3127 | 0.0046 | -0.0028 |
| | | MSE | 0.0002 | 0.0004 | 1.7231 | 0.0482 | 0.0482 |
| | 40 | (0*10,2*15,0*15) | | | | | |
| | | Bias | -0.0094 | -0.0276 | -0.7473 | 2.3503 | 2.3314 |
| | | MSE | 0.0001 | 0.0008 | 0.5585 | 5.5720 | 0.0482 |
| | 40 | (30,0*39) | | | | | |
| | | Bias | -0.0027 | -0.0730 | -0.1854 | 2.8241 | 2.8059 |
| | | MSE | 0.0000 | 0.0053 | 0.0344 | 8.0237 | 0.0482 |
| 70 | 50 | (0*49,20) | | | | | |
| | | Bias | -0.0020 | -0.0148 | -0.9359 | -0.0020 | -0.0045 |
| | | MSE | 0.0000 | 0.0002 | 0.8759 | 0.0432 | 0.0432 |
| | 50 | (0*20,2*10,0*20) | | | | | |
| | | Bias | -0.0093 | -0.0213 | -0.5268 | 1.1800 | 1.1749 |
| | | MSE | 0.0001 | 0.0005 | 0.2775 | 1.4356 | 0.0432 |
| | 50 | (20,0*49) | | | | | |
| | | Bias | -0.0081 | -0.0561 | -0.1273 | 1.5672 | 1.5622 |
| | | MSE | 0.0001 | 0.0032 | 0.0162 | 2.4993 | 0.0432 |
| 90 | 50 | (0*49,40) | | | | | |
| | | Bias | -0.0120 | -0.0179 | -1.3227 | 0.0062 | -0.0002 |
| | | MSE | 0.0001 | 0.0003 | 1.7496 | 0.0385 | 0.0384 |
| | 50 | (0*15,2*20,0*15) | | | | | |
| | | Bias | -0.0150 | -0.0156 | -0.8236 | 2.5062 | 2.4892 |
| | | MSE | 0.0002 | 0.0002 | 0.6784 | 6.3193 | 0.0384 |
| 90 | 60 | (0*59,30) | | | | | |

| N | m | Scheme | MLE | Lindley | I.S | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| | | Bias | -0.0057 | -0.0175 | -1.0327 | 0.0018 | -0.0010 |
| | | MSE | 0.0000 | 0.0003 | 1.0664 | 0.0346 | 0.0346 |
| | 60 | (0*20,2*15,0*25) | | | | | |
| | | Bias | -0.0045 | -0.0221 | -0.5478 | 1.6323 | 1.6258 |
| | | MSE | 0.0000 | 0.0005 | 0.3001 | 2.6990 | 0.0346 |
| | 60 | (30,0*59) | | | | | |
| | | Bias | 0.0012 | -0.0510 | -0.1158 | 2.0324 | 2.0260 |
| | | MSE | 0.0000 | 0.0026 | 0.0134 | 4.1650 | 0.0346 |

**Table 3. Results of Simulation for parameter μ with GLD (α =0.5, $\mu = 0, , \sigma$ =1)**

| N | m | Scheme | MLE | Bayesian Lindley’s | Importance Sampling | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| 50 | 30 | (0*29,20) | | | | | |
| | | Bias | 0.0155 | -0.0507 | -0.3528 | -0.0283 | -0.0219 |
| | | MSE | 0.0002 | 0.0026 | 0.1245 | 0.0997 | 0.0989 |
| | 30 | (0*10,2*10,0*10) | | | | | |
| | | Bias | -0.0015 | -0.0836 | 0.3704 | 0.8626 | 0.8792 |
| | | MSE | 0.0000 | 0.0070 | 0.1372 | 0.8430 | 0.0989 |
| | 30 | (20,0*29) | | | | | |
| | | Bias | 0.0007 | -0.2832 | 1.1404 | 1.6587 | 1.6758 |
| | | MSE | 0.0000 | 0.0802 | 1.3005 | 2.8502 | 0.0989 |
| 50 | 40 | (0*39,10) | | | | | |
| | | Bias | 0.0140 | -0.0257 | 0.3215 | -0.0389 | -0.0319 |
| | | MSE | 0.0002 | 0.0007 | 0.1033 | 0.1003 | 0.0987 |
| | 40 | (0*15,1*10,0*15) | | | | | |
| | | Bias | 0.0081 | -0.1002 | 0.8464 | 0.0444 | 0.0564 |
| | | MSE | 0.0001 | 0.0100 | 0.7164 | 0.1007 | 0.0987 |
| | 40 | (10,0*39) | | | | | |
| | | Bias | 0.0062 | -0.2277 | 1.2132 | 0.4070 | 0.4193 |
| | | MSE | 0.0000 | 0.0519 | 1.4719 | 0.2644 | 0.0987 |
| 70 | 40 | (0*39,30) | | | | | |
| | | Bias | 0.0072 | -0.0312 | -0.4076 | -0.0225 | -0.0183 |
| | | MSE | 0.0001 | 0.0010 | 0.1661 | 0.0720 | 0.0715 |
| | 40 | (0*10,2*15,0*15) | | | | | |
| | | Bias | -0.0026 | -0.0649 | 0.4517 | 1.2506 | 1.2631 |
| | | MSE | 0.0000 | 0.0042 | 0.2040 | 1.6354 | 0.0715 |
| | 40 | (30,0*39) | | | | | |
| | | Bias | 0.0013 | -0.2201 | 1.1894 | 2.0300 | 2.0426 |
| | | MSE | 0.0000 | 0.0484 | 1.4147 | 4.1924 | 0.0715 |
| 70 | 50 | (0*49,20) | | | | | |
| | | Bias | 0.0022 | -0.0221 | 0.0621 | -0.0313 | -0.0263 |
| | | MSE | 0.0000 | 0.0005 | 0.0039 | 0.0723 | 0.0713 |
| | 50 | (0*20,2*10,0*20) | | | | | |
| | | Bias | 0.0092 | -0.0650 | 0.7188 | 0.3066 | 0.3177 |
| | | MSE | 0.0001 | 0.0042 | 0.5167 | 0.1653 | 0.0713 |
| | 50 | (20,0*49) | | | | | |
| | | Bias | 0.0082 | -0.1819 | 1.2491 | 0.8419 | 0.8532 |
| | | MSE | 0.0001 | 0.0331 | 1.5603 | 0.7801 | 0.0713 |
| 90 | 50 | (0*49,40) | | | | | |
| | | Bias | 0.0094 | -0.0294 | -0.4368 | -0.0169 | -0.0138 |
| | | MSE | 0.0001 | 0.0009 | 0.1908 | 0.0563 | 0.0560 |
| | 50 | (0*15,2*20,0*15) | | | | | |
| | | Bias | 0.0023 | -0.0443 | 0.3366 | 1.3371 | 1.3468 |
| | | MSE | 0.0000 | 0.0020 | 0.1133 | 1.8439 | 0.0560 |
| | 50 | (40,0*49) | | | | | |

| N | m | Scheme | MLE | Bayesian Lindley's | Importance Sampling | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| | | Bias | 0.0066 | -0.1864 | 1.2254 | 2.2811 | 2.2910 |
| | | MSE | 0.0000 | 0.0348 | 1.5017 | 5.2593 | 0.0560 |
| 90 | 60 | (0*59,30) | | | | | |
| | | Bias | 0.0086 | -0.0152 | -0.0725 | -0.0217 | -0.0178 |
| | | MSE | 0.0001 | 0.0002 | 0.0053 | 0.0563 | 0.0558 |
| | 60 | (0*20,2*15,0*25) | | | | | |
| | | Bias | 0.0041 | -0.0531 | 0.6870 | 0.5890 | 0.5989 |
| | | MSE | 0.0000 | 0.0028 | 0.4719 | 0.4027 | 0.0558 |
| | 60 | (30,0*59) | | | | | |
| | | Bias | 0.0071 | -0.1501 | 1.2685 | 1.1942 | 1.2042 |
| | | MSE | 0.0001 | 0.0225 | 1.6090 | 1.4820 | 0.0558 |

**Table 4. Results of Simulation for parameter $\sigma$ with GLD ($\alpha$ =1.5, $\mu$ = 0, , $\sigma$ =1)**

| N | m | Scheme | MLE | Bayesian Lindley's | Importance Sampling | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| 50 | 30 | (0*29,20) | | | | | |
| | | Bias | -0.0289 | -0.0009 | 0.3606 | 0.0558 | 0.0291 |
| | | MSE | 0.0008 | 0.0000 | 0.1300 | 0.0290 | 0.0253 |
| | 30 | (0*10,2*10,0*10) | | | | | |
| | | Bias | -0.0211 | -0.0069 | 0.0971 | 1.2428 | 1.1861 |
| | | MSE | 0.0004 | 0.0000 | 0.0094 | 1.5704 | 0.0253 |
| | 30 | (20,0*29) | | | | | |
| | | Bias | -0.0154 | 0.0060 | 0.0508 | 1.1522 | 1.0979 |
| | | MSE | 0.0002 | 0.0000 | 0.0026 | 1.3535 | 0.0253 |
| 50 | 40 | (0*39,10) | | | | | |
| | | Bias | -0.0190 | 0.0063 | 0.1550 | 0.0460 | 0.0278 |
| | | MSE | 0.0004 | 0.0000 | 0.0240 | 0.0198 | 0.0174 |
| | 40 | (0*15,1*10,0*15) | | | | | |
| | | Bias | -0.0152 | 0.0001 | 0.0689 | 0.6908 | 0.6614 |
| | | MSE | 0.0002 | 0.0000 | 0.0047 | 0.4949 | 0.0174 |
| | 40 | (10,0*39) | | | | | |
| | | Bias | -0.0134 | 0.0010 | 0.0526 | 0.6559 | 0.6272 |
| | | MSE | 0.0002 | 0.0000 | 0.0028 | 0.4479 | 0.0174 |
| 70 | 40 | (0*39,30) | | | | | |
| | | Bias | -0.0189 | -0.0043 | 0.3667 | 0.0448 | 0.0247 |
| | | MSE | 0.0004 | 0.0000 | 0.1345 | 0.0216 | 0.0192 |
| | 40 | (0*10,2*15,0*15) | | | | | |
| | | Bias | -0.0154 | -0.0017 | 0.0614 | 1.4195 | 1.3730 |
| | | MSE | 0.0002 | 0.0000 | 0.0038 | 2.0347 | 0.0192 |
| 70 | 50 | (0*49,20) | | | | | |
| | | Bias | -0.0153 | 0.0000 | 0.2044 | 0.0359 | 0.0210 |
| | | MSE | 0.0002 | 0.0000 | 0.0418 | 0.0159 | 0.0144 |
| | 50 | (0*20,2*10,0*20) | | | | | |
| | | Bias | -0.0126 | 0.0015 | 0.0639 | 0.9904 | 0.9617 |
| | | MSE | 0.0002 | 0.0000 | 0.0041 | 0.9955 | 0.0144 |
| | 50 | (20,0*49) | | | | | |
| | | Bias | -0.0100 | 0.0015 | 0.0413 | 0.9326 | 0.9047 |
| | | MSE | 0.0001 | 0.0000 | 0.0017 | 0.8843 | 0.0144 |
| 90 | 50 | (0*49,40) | | | | | |
| | | Bias | -0.0178 | -0.0025 | 0.3658 | 0.0389 | 0.0228 |
| | | MSE | 0.0003 | 0.0000 | 0.1338 | 0.0173 | 0.0228 |
| | 50 | (0*15,2*20,0*15) | | | | | |
| | | Bias | -0.0108 | -0.0062 | 0.0843 | 1.5284 | 1.4892 |
| | | MSE | 0.0001 | 0.0000 | 0.0071 | 2.3518 | 0.0155 |

| N | m | Scheme | MLE | Bayesian Lindley's | Importance Sampling | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| 90 | 60 | (0*59,30) | | | | | |
| | | Bias | -0.0115 | -0.0008 | 0.2394 | 0.0315 | 0.0188 |
| | | MSE | 0.0001 | 0.0000 | 0.0573 | 0.0134 | 0.0123 |
| | 60 | (0*20,2*15,0*25) | | | | | |
| | | Bias | -0.0092 | -0.0006 | 0.0529 | 1.2133 | 1.1860 |
| | | MSE | 0.0001 | 0.0000 | 0.0028 | 1.4845 | 0.0123 |
| | 60 | (30,0*59) | | | | | |
| | | Bias | -0.0121 | 0.0044 | 0.0405 | 1.1111 | 1.0851 |
| | | MSE | 0.0001 | 0.0000 | 0.0016 | 1.2469 | 0.0123 |

**Table 5. Results of Simulation for parameter $\sigma$ with GLD (α =1.0, $\mu$ = 0, , $\sigma$ =1)**

| N | m | Scheme | MLE | Lindley | I.S | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| 50 | 30 | (0*29,20) | | | | | |
| | | Bias | -0.0256 | 0.0105 | 0.1913 | 0.0559 | 0.0298 |
| | | MSE | 0.0007 | 0.0001 | 0.0366 | 0.0285 | 0.0247 |
| | 30 | (0*10,2*10,0*10) | | | | | |
| | | Bias | -0.0189 | -0.0015 | 0.0560 | 1.4334 | 1.3733 |
| | | MSE | 0.0004 | 0.0000 | 0.0031 | 2.0801 | 0.0247 |
| | 30 | (20,0*29) | | | | | |
| | | Bias | -0.0144 | -0.0049 | 0.0532 | 1.3737 | 1.3151 |
| | | MSE | 0.0002 | 0.0000 | 0.0028 | 1.9125 | 0.0247 |
| 50 | 40 | (0*39,10) | | | | | |
| | | Bias | -0.0150 | 0.0064 | 0.0746 | 0.0473 | 0.0292 |
| | | MSE | 0.0002 | 0.0000 | 0.0056 | 0.0199 | 0.0173 |
| | 40 | (0*15,1*10,0*15) | | | | | |
| | | Bias | -0.0160 | 0.0019 | 0.0416 | 0.7485 | 0.7182 |
| | | MSE | 0.0003 | 0.0000 | 0.0017 | 0.5779 | 0.0173 |
| | 40 | (10,0*39) | | | | | |
| | | Bias | -0.0103 | -0.0013 | 0.0398 | 0.7399 | 0.7098 |
| | | MSE | 0.0001 | 0.0000 | 0.0016 | 0.5651 | 0.0173 |
| 70 | 40 | (0*39,30) | | | | | |
| | | Bias | -0.0173 | 0.0067 | 0.1925 | 0.0424 | 0.0228 |
| | | MSE | 0.0003 | 0.0000 | 0.0371 | 0.0209 | 0.0188 |
| | 40 | (0*10,2*15,0*15) | | | | | |
| | | Bias | -0.0161 | -0.0015 | 0.0332 | 1.6443 | 1.5946 |
| | | MSE | 0.0003 | 0.0000 | 0.0011 | 2.7228 | 0.0188 |
| | 40 | (30,0*39) | | | | | |
| | | Bias | -0.0091 | -0.0003 | 0.0343 | 1.5484 | 1.5005 |
| | | MSE | 0.0001 | 0.0000 | 0.0012 | 2.4167 | 0.0188 |
| 70 | 50 | (0*49,20) | | | | | |
| | | Bias | -0.0130 | 0.0095 | 0.0982 | 0.0349 | 0.0202 |
| | | MSE | 0.0002 | 0.0001 | 0.0096 | 0.0157 | 0.0142 |
| | 50 | (0*20,2*10,0*20) | | | | | |
| | | Bias | -0.0115 | 0.0011 | 0.0292 | 1.1164 | 1.0863 |
| | | MSE | 0.0001 | 0.0000 | 0.0009 | 1.2608 | 0.0142 |
| | 50 | (20,0*49) | | | | | |
| | | Bias | -0.0088 | -0.0008 | 0.0325 | 1.0805 | 1.0509 |
| | | MSE | 0.0001 | 0.0000 | 0.0011 | 1.1820 | 0.0142 |
| 90 | 50 | (0*49,40) | | | | | |
| | | Bias | -0.0149 | 0.0006 | 0.1943 | 0.0357 | 0.0200 |
| | | MSE | 0.0002 | 0.0000 | 0.0378 | 0.0167 | 0.0152 |
| | 50 | (0*15,2*20,0*15) | | | | | |
| | | Bias | -0.0129 | 0.0017 | 0.0374 | 1.7541 | 1.7123 |
| | | MSE | 0.0002 | 0.0000 | 0.0014 | 3.0922 | 0.0152 |

| N | m | Scheme | MLE | Lindley | I.S | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| 90 | 60 | (0*59,30) | | | | | |
| | | Bias | -0.0126 | 0.0030 | 0.1154 | 0.0308 | 0.0183 |
| | | MSE | 0.0002 | 0.0000 | 0.0133 | 0.0132 | 0.0121 |
| | 60 | (0*20,2*15,0*25) | | | | | |
| | | Bias | -0.0100 | -0.0007 | 0.0269 | 1.3707 | 1.3420 |
| | | MSE | 0.0001 | 0.0000 | 0.0007 | 1.8909 | 0.0121 |
| | 60 | (30,0*59) | | | | | |
| | | Bias | -0.0081 | 0.0004 | 0.0262 | 1.3090 | 1.2812 |
| | | MSE | 0.0001 | 0.0000 | 0.0007 | 1.7258 | 0.0121 |

**Table 6. Results of Simulation for parameter $\sigma$ with GLD ($\alpha$ =0.5, $\mu$ = 0, , $\sigma$ =1)**

| N | M | Scheme | MLE | Bayesian Lindley's | Importance Sampling | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| 50 | 30 | (0*29,20) | | | | | |
| | | Bias | -0.0206 | 0.0537 | 0.0684 | 0.0528 | 1.0274 |
| | | MSE | 0.0004 | 0.0029 | 0.0047 | 0.0275 | 0.0241 |
| | 30 | (0*10,2*10,0*10) | | | | | |
| | | Bias | -0.0170 | -0.0005 | 0.0779 | 1.7266 | 1.6609 |
| | | MSE | 0.0003 | 0.0000 | 0.0061 | 3.0060 | 0.0241 |
| | 30 | (20,0*29) | | | | | |
| | | Bias | -0.0151 | -0.0060 | 0.1052 | 1.8265 | 1.7584 |
| | | MSE | 0.0002 | 0.0000 | 0.0111 | 3.3607 | 0.0241 |
| 50 | 40 | (0*39,10) | | | | | |
| | | Bias | -0.0124 | 0.0022 | 0.0422 | 0.0506 | -0.0319 |
| | | MSE | 0.0002 | 0.0000 | 0.0018 | 0.0208 | 0.0179 |
| | 40 | (0*15,1*10,0*15) | | | | | |
| | | Bias | -0.0169 | 0.0018 | 0.0696 | 0.8021 | 0.7697 |
| | | MSE | 0.0003 | 0.0000 | 0.0048 | 0.6616 | 0.0179 |
| | 40 | (10,0*39) | | | | | |
| | | Bias | -0.0132 | -0.0071 | 0.0963 | 0.8504 | 0.8172 |
| | | MSE | 0.0002 | 0.0001 | 0.0093 | 0.7414 | 0.0179 |
| 70 | 40 | (0*39,30) | | | | | |
| | | Bias | -0.0189 | 0.0416 | 0.0590 | 0.0466 | 0.0275 |
| | | MSE | 0.0004 | 0.0017 | 0.0035 | 0.0207 | 0.0182 |
| | 40 | (0*10,2*15,0*15) | | | | | |
| | | Bias | -0.0140 | -0.0017 | 0.0670 | 2.0821 | 2.0260 |
| | | MSE | 0.0002 | 0.0000 | 0.0045 | 4.3539 | 0.0182 |
| | 40 | (30,0*39) | | | | | |
| | | Bias | -0.0121 | -0.0085 | 0.0948 | 2.1116 | 2.0549 |
| | | MSE | 0.0001 | 0.0001 | 0.0090 | 4.4772 | 0.0182 |
| 70 | 50 | (0*49,20) | | | | | |
| | | Bias | -0.0093 | 0.0114 | 0.0332 | 0.0383 | 0.0234 |
| | | MSE | 0.0001 | 0.0001 | 0.0011 | 0.0160 | 0.0143 |
| | 50 | (0*20,2*10,0*20) | | | | | |
| | | Bias | -0.0113 | 0.0030 | 0.0657 | 1.2792 | 1.2465 |
| | | MSE | 0.0001 | 0.0000 | 0.0043 | 1.6509 | 0.0143 |
| | 50 | (20,0*49) | | | | | |
| | | Bias | -0.0106 | -0.0089 | 0.0832 | 1.3285 | 1.2951 |
| | | MSE | 0.0001 | 0.0001 | 0.0069 | 1.7796 | 0.0143 |
| 90 | 50 | (0*49,40) | | | | | |
| | | Bias | -0.0146 | 0.0334 | 0.0548 | 0.0354 | 0.0202 |

| N | M | Scheme | MLE | Bayesian Lindley's | Importance Sampling | BLUE | BLEE |
|---|---|---|---|---|---|---|---|
| | | MSE | 0.0002 | 0.0011 | 0.0030 | 0.0161 | 0.0147 |
| | 50 | (0*15,2*20,0*15) | | | | | |
| | | Bias | -0.0134 | -0.0001 | 0.0459 | 2.2139 | 2.1669 |
| | | MSE | 0.0002 | 0.0000 | 0.0021 | 4.9164 | 0.0147 |
| | 50 | (40,0*49) | | | | | |
| | | Bias | -0.0081 | -0.0030 | 0.0860 | 2.3172 | 2.2686 |
| | | MSE | 0.0001 | 0.0000 | 0.0074 | 5.3844 | 0.0147 |
| 90 | 60 | (0*59,30) | | | | | |
| | | Bias | -0.0121 | 0.0179 | 0.0277 | 0.0312 | 0.0188 |
| | | MSE | 0.0001 | 0.0003 | 0.0008 | 0.0131 | 0.0120 |
| | 60 | (0*20,2*15,0*25) | | | | | |
| | | Bias | -0.0076 | -0.0008 | 0.0602 | 1.6402 | 1.6085 |
| | | MSE | 0.0001 | 0.0000 | 0.0036 | 2.7023 | 0.0120 |
| | 60 | (30,0*59) | | | | | |
| | | Bias | -0.0088 | -0.0036 | 0.0773 | 1.6694 | 1.6373 |
| | | MSE | 0.0001 | 0.0000 | 0.0060 | 2.7990 | 0.0120 |

The results given in Tables 1 – 6 Show that the maximum likelihood estimator has the best overall performance in terms of bias and mean squared error. It is followed closely by the Lindley's approximation to the Bayes estimator. The importance sampling estimator does not appear to perform well in our simulations. The approximate BLUE and BLEE estimators have similar performance, however, the approximate BLEE appears to have slightly better performance than the approximate BLUE. But both of them are dominated by the MLE and the Lindley's approximation of the Bayes estimator.

The parameter α does not appear to have any effect on the relative performance of the estimators for the location and scale parameters. However, the biases and MSEs of the estimators tend to decrease for smaller values of $\alpha$.

# 5 Real Data Example: Breakdown of an Insulating Fluid

To evaluate and analyze the quality of transformers and their insulating fluids, a variety of tests has been devised. To explain this, for example, let's consider the Dielectric Breakdown Test, which assesses an insulating liquid's capacity to endure electrical stress up to the point of failure. It displays the voltage at which there will be a breakdown. Moisture, dirt, and conductive particle contamination will induce failure at levels below what is considered tolerable. Nelson [10] provided a data for the breakdown of an insulating fluid testing experiment. This data collection was examined and evaluated by Balakrishnan and Hossain [6] examining Type II generalized logistic distribution inference under progressive Type II censoring. Balakrishnan and Hossain evaluated and examined the data set that fits the Type II Generalized Logistic Distribution and finding out that MLE and Approximate MLE are very close in the inferencing. In this example n= 19 and m=8 with $\alpha$ =1. The data and the results are shown in Tables 7 and 8.

**Table 7. Insulating Fluid Data**

| I | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $x_i$ | -1.6608 | -0.2485 | -0.0409 | 0.2700 | 1.0224 | 1.5789 | 1.8718 | 1.9947 |
| $r_i$ | 0 | 0 | 3 | 0 | 3 | 0 | 0 | 5 |

**Table 8. Parameter Estimates Based on Insulating Fluid Data**

| Estimator | $\sigma$ | $\mu$ |
|---|---|---|
| MLE | 0.9027 | 1.8757 |
| Bayesian – Lindley's Approach | 0.9716 | 1.8511 |
| Bayesian – Importance Sampling | 1.4455 | -0.2370 |
| Importance Sampling | | |
| BLUE | 1.4211 | 2.5867 |
| BLEE | 1.2786 | 2.4809 |

The results show that the MLE and the Bayes estimator based on Lindley's approximation are close to each other and somewhat smaller than the linear estimators. Based on our simulation study, the former estimators are more precise and reliable.

# 6. Summary and Conclusion

In this study, based on progressively type II censored data, we considered point estimation of location and scale parameters in type II Generalized Logistic Distribution (Type II GLD). We developed three estimators (ABLUE and ABLEE and Importance Sampling Estimator) for the unknown parameters. We also included the maximum likelihood estimators (MLE) and Bayes estimators approximated by the Lindley's Approach for comparison purposes.

The results of the simulation study reveal that MLE and Lindley's approximation to the Bayes estimator perform better than the other estimators developed in this paper. They have the smallest bias and MSE values as shown during the simulation study. As for the effect of the parameter $\alpha$ value on the location and scale estimator's bias and MSE values, estimators got better results for smaller values of $\alpha$.

The conclusion of this work is that the MLE has the overall best performance for estimating the parameters of the type II generalized logistic distribution. However, for small sample sizes, numerical problems can occur. In such situations, the approximate linear estimators like the ABLUE and ABLEE can provide a viable alternative. The Bayes estimator performs very well too, especially the approximation based on Lindley's approach.

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# Competing Interests

Authors have declared that no competing interests exist.

# References

[1] Balakrishnan N, Sandhu RA. A Simple Simulational Algorithm for Generating Progressive Type-II Censored Samples, The American Statistician. 1995;49(2):229-230.

[2] Salah MM. On progressive Type-II censored samples from alpha power exponential distribution. Journal of Mathematics. 2020;Article ID 2584184. Available: https://doi.org/10.1155/2020/2584184

[3] Nassar MM, Elmasry A. A study of generalized logistic distributions, Journal of the Egyptian Mathematical Society. 2012;20(2):126 – 133.

[4] Azizpour M, Asgharzadeh A. Inference for the Type-II Generalized Logistic Distribution with Progressive Hybrid Censoring. Journal of Statistical Research of Iran. 2018;14 (2):189-217.

[5] Aljarrah, M., Fayome, F. and Lee, C. Generalized logistic distribution and its regression model. Journal of Statistical Distributions and Applications. 2020: 7(7). Available: https://doi.org/10.1186/s40488-020-00107-8.

[6] Balakrishnan N, Hossain A. Inference for the Type II generalized logistic distribution under progressive Type II censoring. Journal of Statistical Computation and Simulation. 2007;77(12): 1013 – 1031.

[7] Rimawi, Rana and Baklizi, A. Estimation in the Type II Generalized Logistic Distribution Based on Progressively Type II Censored Data. Proceedings of the GSRD International Conference. 2021;28 – 35.

[8] Balakrishnan N, Cramer E. The art of progressive censoring, Application to Reliability and Quality. Birkhauser; 2014.

[9] Balakrishnan N, Sandhu RA. Best Linear Unbiased and Maximum Likelihood Estimation for Exponential Distributions under General Progressive Type-II Censored Samples, the Indian Journal of Statistics, Series B. 1996;58(1): 1-9.

[10] Nelson W. Applied Life Data Analysis. New York: John Wiley and Sons; 1982.

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