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Rogue Waves of the Lakshmanan Porsezian Daniel Equation Depending on Multi-parameters

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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ABSTRACT

Quasi-rational solutions to the Lakshmanan Porsezian Daniel equation are presented. We construct explicit expressions of these solutions for the first orders depending on real parameters.
We study the patterns of these configurations in the (x, t) plane in function of the different parameters.
We observe in the case of order 2, three rogue waves which move according to the two parameters.
In the case of order 3, six rogue waves are observed with specific configurations moving according to the four parameters.

Keywords: Lakshmanan Porsezian Daniel equation; quasi-rational solutions.

PACS numbers: 33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd.

1 INTRODUCTION

We consider the Lakshmanan Porsezian Daniel (LPD) equation in the following normalization

$$iu_t + u_{xxxx} + 8|u|^2 u_{xx} + 2u^2 \overline{u}_{xx} + 6u_x^2 \overline{u} + 4u|u_x|^2 + 6|u|^4 u = 0, \quad (1.1)$$

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where the subscripts mean the partial derivatives.

There are a different models that describe the dynamics of soliton propagation. One of the most important models is the nonlinear Lakshmanan Porsezian Daniel (LPD) equation. This equation with spatio-temporal dispersion as well as group velocity dispersion, is used to get and describe solitons. This model was introduced originally in the context of Heisenberg spin chain [1]. Solitons are some of the main focal points in the field of mathematical physics, optical fibers and nonlinear optics in particular. These solutions are self-reinforcing single waves, which move at a constant velocity while maintaining its shape; they are relevant because of their localized and stable nature. The LPD equation has been studied in the polarization-preserving fibers context or in the context of the birefringent fibers [2]; it has been used in the study of the propagation of periodic ultrashort pulses in the optical fibers [3]. Many mathematical methods have used to study this equation; we can mention the following ones. For instance, the method of undetermined coefficients is applied in [4] to look for bright, dark and singular soliton solutions. The extended trial equation method has been used to construct bright solitons, dark solitons, periodic solitary wave, rational and elliptic solutions in [5]. The semi inverse variational principle has been employed in [6] to construct bright solitons with Kerr and power laws of nonlinearity. One can also quote the extended Jacobi elliptic function approach [7] to obtain dark and singular optical solitons; the $\exp(-\phi(\epsilon))$-expansion method in [8]; the Lie symmetry analysis in [9]; the Riccati equation method has been applied to get dark and bright soliton solutions of (1) in [10]; the sine-Gordon expansion method has been used to get analytical solutions of the LPD equation in [11]; the modified extended direct algebraic method [12] and expansion method [13].

2 QUASI-RATIONAL SOLUTIONS OF ORDER 1 TO THE LAKSHMANAN PORSEZIAN DANIEL EQUATION

The function $v(x,t)$ defined by

$$v(x,t)=\left(1-4\,\frac{1+24\,it}{1+4\,x^2+576\,t^2}\right)\mathrm{e}^{6\,it} \tag{2.1}$$

is a solution to the Lakshmanan Porsezian Daniel equation (1.1)

(2.2)

Proof: We have to replace the expression of the solution given by (2.1) and check that (1.1) is verified.

3 QUASI-RATIONAL SOLUTIONS OF ORDER 2 TO THE LAKSHMANAN PORSEZIAN DANIEL EQUATION

The function $v(x,t)$ defined by

$$v(x,t)=\left(1-12\frac{n(x,t)}{d(x,t)}\right)\exp(6it+2a_1) \tag{3.1}$$

with

$n(x,t)=(2\,x-12\,b_1)^4+6\,((4\,a_1+24\,t)^2+1)(2\,x-12\,b_1)^2-192\,b_1(2\,x-12\,b_1)+5\,(4\,a_1+24\,t)^4+$

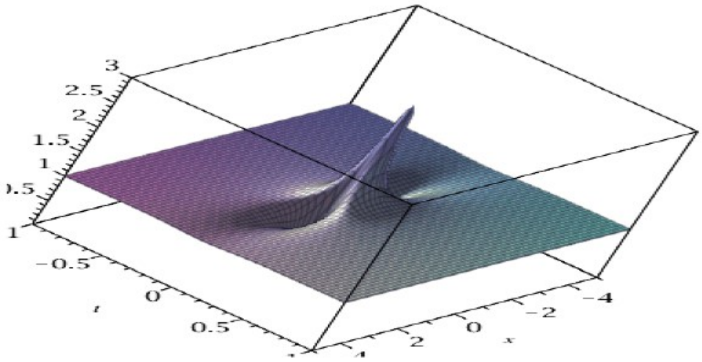

Fig. 1. Solution of order 1 **to the equation** (1.1)

$18\,(4\,a_1+24\,t)^2+384\,t(4\,a_1+24\,t)-3+i((4\,a_1+24\,t)(2\,x-12\,b_1)^4+2\,((4\,a_1+24\,t)^3-12\,a_1-168\,t)(2\,x-12\,b_1)^2-192\,(4\,a_1+24\,t)b_1(2\,x-12\,b_1)+(4\,a_1+24\,t)^5+2\,(4\,a_1+24\,t)^3+192\,t(4\,a_1+24\,t)^2-60\,a_1-552\,t)$
and
$d(x,t) = ((2\,x-12\,b_1)^2+(4\,a_1+24\,t)^2+1)^3+192\,b_1(2\,x-12\,b_1)^3-24\,((4\,a_1+24\,t)^2+48\,t(4\,a_1+24\,t)-1)(2\,x-12\,b_1)^2-576\,((4\,a_1+24\,t)^2+1)b_1(2\,x-12\,b_1)+24\,(4\,a_1+24\,t)^4+384\,t(4\,a_1+24\,t)^3+96\,(4\,a_1+24\,t)^2+3456\,t(4\,a_1+24\,t)+36864\,t^2+9216\,{b_1}^2+8$
is a solution to the Lakshmanan Porsezian Daniel equation (1.1).

Proof: It is sufficient to replace the expression of the solution given by (3.1) and check that the relation (1.1) is verified.

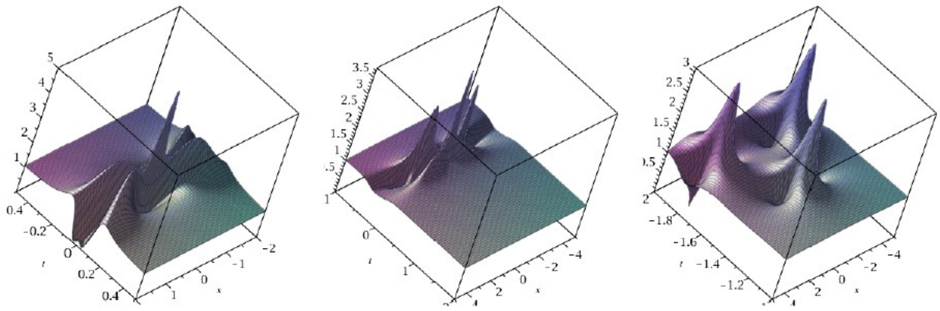

Fig. 2. Solution of order 1 **to the equation** (1.1)**; to the left** $a_1 = 0$**,** $b_1 = 0$**; in the center** $a_1 = 1$**,** $b_1 = 0$**; to the right** $a_1 = 10$**,** $b_1 = 0$

A very fast evolution of the structure of the solution is observed when the parameter a_1 grows; it evolves toward three peaks.

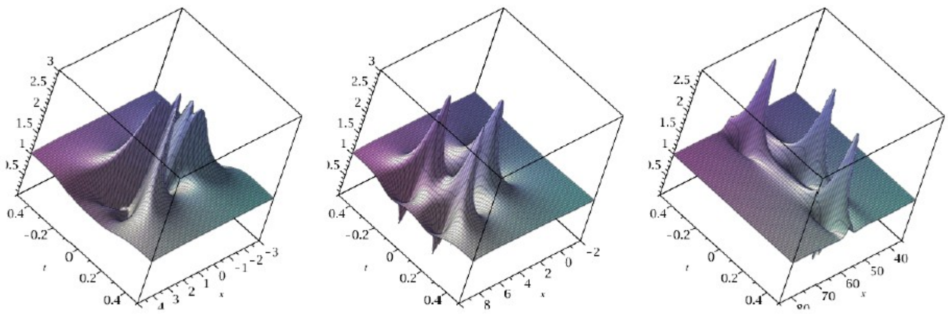

Fig. 3. Solution of order 1 to the equation (1.1); to the left $a_1 = 0$, $b_1 = 0.1$; in the center $a_1 = 0$, $b_1 = 1$; to the right $a_1 = 0$, $b_1 = 10$

As in the case of the parameter a_1, an evolution of the structure of the solution toward three peaks is observed when the parameter b_1 grows.

4 QUASI-RATIONAL SOLUTIONS OF ORDER 3 TO THE LAKSHMANAN PORSEZIAN DANIEL EQUATION

The function $v(x,t)$ defined by

$$v(x,t) = \left(1 - 24\frac{n(x,t)}{d(x,t)}\right) \mathrm{e}^{6\,i\alpha\,t} \tag{4.1}$$

with

$n(x,t) = 1024\,x^{10} + 3840\,(1 + 576\,t^2)x^8 + 64\,(210 - 218880\,t^2 + 16588800\,t^4)x^6 + 675 + 16\,(-1584000\,t^2 - 315187200\,t^4 + 13377208320\,t^6 - 450)x^4 + 4\,(5424537600\,t^4 + 80263249920\,t^6 + 4953389137920\,t^8 - 1555200\,t^2(5 - 3072\,t^2) - 675 - 19353600\,t^2)x^2 + 1977915801600\,t^6 + 107323431321600\,t^8 + 697437190619136\,t^{10} + 149299200\,t^4(-17 + 3072\,t^2) + 2388787200\,t^4 + 259200\,t(16\,t - 16384\,t^3) + 388800\,t^2(-3 + 4096\,t^2) + 23500800\,t^2 + i(24576\,tx^{10} + 256\,(-840\,t + 69120\,t^3)x^8 + 64\,(240\,t - 4147200\,t^3 + 79626240\,t^5)x^6 + 16\,(-22809600\,t^3 - 3264675840\,t^5 + 45864714240\,t^7 + 10800\,t(-3 - 1024\,t^2) + 14400\,t)x^4 + 4\,(51836682240\,t^5 - 275188285440\,t^7 + 13209037701120\,t^9 - 12441600\,t^3(7 - 3072\,t^2) + 16200\,t(7 + 4096\,t^2) + 345600\,t + 88473600\,t^3)x^2 + 716636160\,t^5(-107 + 3072\,t^2) + 3110400\,t^2(-176\,t - 16384\,t^3) - 3110400\,t^3(11 + 20480\,t^2) - 16200\,t(-7 + 22528\,t^2) - 265420800\,t^3 - 138549657600\,t^5 - 2155641569280\,t^7 + 224553640919040\,t^9 + 1521681143169024\,t^{11} + 151200\,t)$

and

$d(x,t) = 2024 + 256\,(-207360\,t^2 + 120)x^8 + 64\,(3778560\,t^2 - 291962880\,t^4 + 2320)x^6 + 16\,(-5255331840\,t^4 - 91729428480\,t^6 + 138240\,t^2(56 - 11520\,t^2) - 26265600\,t^2 + 3360)x^4 + 4\,(5870683422720\,t^6 + 26418075402240\,t^8 - 79626240\,t^4(-326 - 34560\,t^2) + 276480\,t^2(-76 + 311040\,t^2) + 57330892800\,t^4 - 1555200\,t(96\,t - 16384\,t^3) + 12144 + 8294400\,t^2)x^2 + 15288238080\,t^6(191 + 6912\,t^2) + 298598400\,t^3(368\,t - 16384\,t^3) + 79626240\,t^4(599 - 57600\,t^2) - 388800\,t(-496\,t - 65536\,t^3) + 13824\,t^2(3881 + 7257600\,t^2) + (1 + 4\,x^2 + 576\,t^2)^6 + 223948800\,t^2 + 33973862400\,t^4 + 4127824281600\,t^6 + 937841676779520\,t^8 + 12680676193075200\,t^{10}$

is a solution to the (LPD) equation (1.1).

Proof: We check that the relation (1.1) is verified when we replace the expression of the solution given by (5.1).

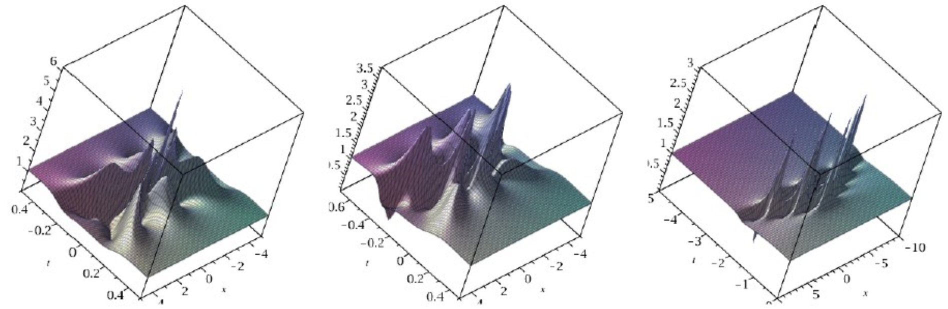

Fig. 4. Solution of order 3 **to the equation** (1.1)**; to the left** $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$**; in the center** $a_1 = 1$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$**; to the right** $a_1 = 10$, $b_1 = 0$, $a_2 = 10$, $b_2 = 0$**; with** $\alpha = \beta = 1$

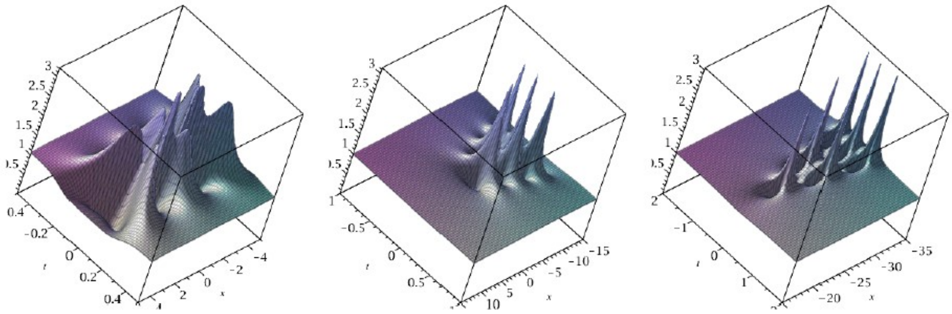

Fig. 5. Solution of order 3 **to the equation** (1.1)**; to the left** $a_1 = 0$, $b_1 = 0.1$, $a_2 = 0$, $b_2 = 0$**; in the center** $a_1 = 0$, $b_1 = 1$, $a_2 = 0$, $b_2 = 0$**; to the right** $a_1 = 0$, $b_1 = 5$, $a_2 = 0$, $b_2 = 0$**; with** $\alpha = \beta = 1$

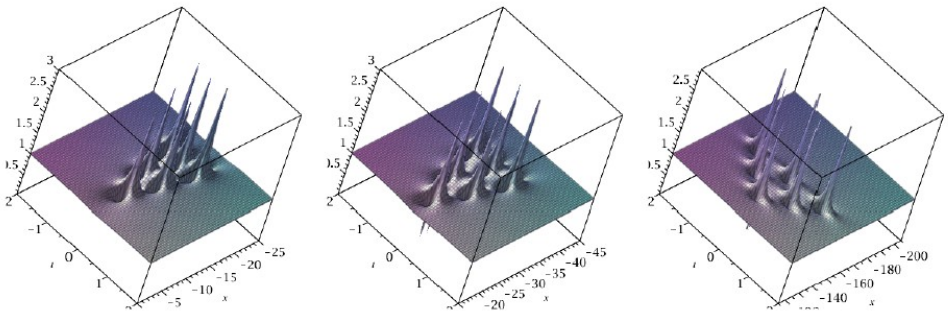

Fig. 6. Solution of order 3 **to the equation** (1.1)**; to the left** $a_1 = 0$, $b_1 = 0$, $a_2 = 0.5$, $b_2 = 0$**; in the center** $a_1 = 0$, $b_1 = 0$, $a_2 = 1$, $b_2 = 0$**; to the right** $a_1 = 2$, $b_1 = 0$, $a_2 = 5$, $b_2 = 0$**; with** $\alpha = \beta = 1$

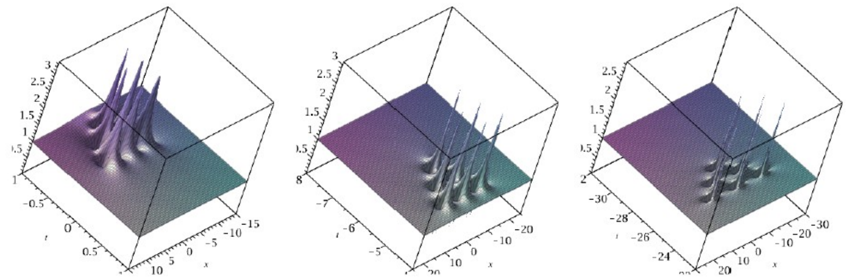

Fig. 7. Solution of order 3 to the equation (1.1); to the left $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0.1$; in the center $a_1 = 0$, $b_1 = 10$, $a_2 = 0$, $b_2 = 1$; to the right $a_1 = 10$, $b_1 = 0$, $a_2 = 0$, $b_2 = 5$; with $\alpha = \beta = 1$

We can give also the solutions to the Lakshmanan Porsezian Daniel equation depending on 4 real parameters. But, because of the length of the expression, we only give it in the appendix.

We give patterns of the modulus of the solutions in function of the parameters a_1, a_2, b_1, b_2.

In the evolution of the structure of the solutions according to the different parameters, we note the appearance of triangles with six peaks with more at least speed according to the parameters.

5 CONCLUSION

Quasi-rational solutions to the Lakshmanan Porsezian Daniel equation have been given for the first orders depending on several real parameters.

In the case of order 2, the evolution of the structure of the solutions toward three peaks is observed when parameters a_1 or b_1 grow.
In the same way, in the case of order 3, the evolution of the structure of the solutions toward six peaks is observed when parameters a_1, a_2, a_1, a_2 grow.

These solutions have to be compared with these constructed by the author in the case of the NLS or mKdV equations [14, 15, 16, 17, 18].

We can cite some recent works about this equation. In [11], new travelling wave solutions to the (LPD) equation with Kerr nonlinearity are built using Bäcklund transformation method based on Riccati equation, Kudryashov method and a new auxiliary ordinary differential equation (ODE). In [20], bright, dark, singular, kink and periodic optical solitons solutions of the LPD equation are constructed by improved $\tan\psi(\eta)/2$-method. In [21], three images of nonlinearity to the fractional LPD equation in birefringent fibers are investigated. In particular, the new bright, periodic wave and singular optical soliton solutions are constructed via the $m + G'/G$ expansion method. In [22], new analytical solutions to the LPD equation is presented by Jacobi elliptic functions. It would be relevant to continue the study of this article to determine other configurations in particular, those concerning no longer triangles but rings as in the case of the NLS equation.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX

Solution of order 3 to the (LPD) equation depending on 4 real parameters :
The function $v(x,t)$ defined by

$$v(x,t) = \left(1 - 24\frac{n(x,t)}{d(x,t)}\right) \mathrm{e}^{i(2\,a_1+6\,t+20\,b_2)} \tag{5.1}$$

with

$$n(x,t) = 675 + (2\,x + 12\,b_1 + 60\,a_2)^{10} + 2190\,(4\,a_1 + 24\,t + 120\,b_2)^6 + 27000\,(-8\,b_1 - 80\,a_2)^2 +
91800\,(-16\,t - 160\,b_2)^2 + 495\,(4\,a_1 + 24\,t + 120\,b_2)^8 + 11\,(4\,a_1 + 24\,t + 120\,b_2)^{10} + 88473600\,{a_2}^2 +
353894400\,{b_2}^2 + 15\,(1 + (4\,a_1 + 24\,t + 120\,b_2)^2)(2\,x + 12\,b_1 + 60\,a_2)^8 + (210 - 60\,(4\,a_1 + 24\,t + 120\,b_2)^2 +
50\,(4\,a_1 + 24\,t + 120\,b_2)^4 + 480\,(4\,a_1 + 24\,t + 120\,b_2)(-16\,t - 160\,b_2))(2\,x + 12\,b_1 + 60\,a_2)^6 + (-720\,(4\,a_1 +
24\,t + 120\,b_2)^2(-8\,b_1 - 80\,a_2) + 5760\,b_1 - 11520\,a_2)(2\,x + 12\,b_1 + 60\,a_2)^5 + (450\,(4\,a_1 + 24\,t + 120\,b_2)^2 -
150\,(4\,a_1 + 24\,t + 120\,b_2)^4 + 70\,(4\,a_1 + 24\,t + 120\,b_2)^6 + 1200\,(4\,a_1 + 24\,t + 120\,b_2)^3(-16\,t - 160\,b_2) -
450 + 5400\,(-8\,b_1 - 80\,a_2)^2 - 1800\,(-16\,t - 160\,b_2)^2 + 3600\,(4\,a_1 + 24\,t + 120\,b_2)(-16\,t - 224\,b_2))(2\,x +
12\,b_1 + 60\,a_2)^4 + (-2400\,(4\,a_1 + 24\,t + 120\,b_2)^4(-8\,b_1 - 80\,a_2) + 28800\,(4\,a_1 + 24\,t + 120\,b_2)(-8\,b_1 -
80\,a_2)(-16\,t - 160\,b_2) - 57600\,b_1 - 806400\,a_2 - 7200\,(4\,a_1 + 24\,t + 120\,b_2)^2(-16\,b_1 - 128\,a_2))(2\,x +
12\,b_1 + 60\,a_2)^3 + (6750\,(4\,a_1 + 24\,t + 120\,b_2)^4 + 420\,(4\,a_1 + 24\,t + 120\,b_2)^6 + 45\,(4\,a_1 + 24\,t + 120\,b_2)^8 -
2700\,(4\,a_1 + 24\,t + 120\,b_2)^2(5 + 4\,(-8\,b_1 - 80\,a_2)^2 - 12\,(-16\,t - 160\,b_2)^2) - 675 - 10800\,(-8\,b_1 -
80\,a_2)^2 - 10800\,(-16\,t - 160\,b_2)^2 + 21600\,(4\,a_1 + 24\,t + 120\,b_2)(-32\,t - 384\,b_2) - 7200\,(4\,a_1 + 24\,t +
120\,b_2)^3(-32\,t - 128\,b_2))(2\,x + 12\,b_1 + 60\,a_2)^2 + (-1680\,(4\,a_1 + 24\,t + 120\,b_2)^6(-8\,b_1 - 80\,a_2) -
28800\,(4\,a_1 + 24\,t + 120\,b_2)^3(-8\,b_1 - 80\,a_2)(-16\,t - 160\,b_2) - 10800\,(4\,a_1 + 24\,t + 120\,b_2)^2(-8\,b_1 -
272\,a_2) - 86400\,b_1 - 1209600\,a_2 + 43200\,(-8\,b_1 - 80\,a_2)^3 + 43200\,(-8\,b_1 - 80\,a_2)(-16\,t - 160\,b_2)^2 +
3600\,(4\,a_1 + 24\,t + 120\,b_2)^4(-8\,b_1 + 80\,a_2) - 86400\,(4\,a_1 + 24\,t + 120\,b_2)((-8\,b_1 - 80\,a_2)(-16\,t - 160\,b_2) +
32\,(-16\,t - 160\,b_2)a_2 - 64\,(-8\,b_1 - 80\,a_2)b_2))(2\,x + 12\,b_1 + 60\,a_2) - 720\,(4\,a_1 + 24\,t + 120\,b_2)^7(-16\,t -
160\,b_2) + 450\,(4\,a_1 + 24\,t + 120\,b_2)^4(-17 + 28\,(-8\,b_1 - 80\,a_2)^2 + 12\,(-16\,t - 160\,b_2)^2) - 3600\,(4\,a_1 +
24\,t + 120\,b_2)^3(-48\,t - 1376\,b_2) - 720\,(4\,a_1 + 24\,t + 120\,b_2)^5(-272\,t - 3168\,b_2) + 10800\,(4\,a_1 + 24\,t +
120\,b_2)(16\,t + 224\,b_2 + 4\,(-8\,b_1 - 80\,a_2)^2(-16\,t - 160\,b_2) + 4\,(-16\,t - 160\,b_2)^3) + 675\,(4\,a_1 + 24\,t +
120\,b_2)^2(-3 + 16\,(-8\,b_1 - 80\,a_2)^2 + 16\,(-16\,t - 160\,b_2)^2 - 4096\,(-8\,b_1 - 80\,a_2)a_2 - 8192\,(-16\,t -
160\,b_2)b_2) - 2764800\,(-8\,b_1 - 80\,a_2)a_2 - 11059200\,(-16\,t - 160\,b_2)b_2 + i(151200\,t - 5529600\,(-8\,b_1 -
80\,a_2)(-16\,t - 160\,b_2)a_2 + 64800\,(-16\,t - 160\,b_2)^3 - 870\,(4\,a_1 + 24\,t + 120\,b_2)^7 + 25\,(4\,a_1 + 24\,t +
120\,b_2)^9 + (4\,a_1 + 24\,t + 120\,b_2)^{11} - 21600\,(-8\,b_1 - 80\,a_2)^2(-16\,t - 160\,b_2) + (4\,a_1 + 24\,t + 120\,b_2)(2\,x +
12\,b_1 + 60\,a_2)^{10} + (-60\,a_1 - 840\,t - 6600\,b_2 + 5\,(4\,a_1 + 24\,t + 120\,b_2)^3)(2\,x + 12\,b_1 + 60\,a_2)^8 + (-600\,a_1 +
240\,t + 58800\,b_2 - 140\,(4\,a_1 + 24\,t + 120\,b_2)^3 + 10\,(4\,a_1 + 24\,t + 120\,b_2)^5 + 240\,(4\,a_1 + 24\,t + 120\,b_2)^2(-16\,t -
160\,b_2))(2\,x + 12\,b_1 + 60\,a_2)^6 + (-240\,(4\,a_1 + 24\,t + 120\,b_2)^3(-8\,b_1 - 80\,a_2) - 1440\,(-8\,b_1 - 80\,a_2)(-16\,t -
160\,b_2) + 720\,(4\,a_1 + 24\,t + 120\,b_2)(-8\,b_1 - 176\,a_2))(2\,x + 12\,b_1 + 60\,a_2)^5 + (-450\,(4\,a_1 + 24\,t + 120\,b_2)^3 -
210\,(4\,a_1 + 24\,t + 120\,b_2)^5 + 10\,(4\,a_1 + 24\,t + 120\,b_2)^7 + 300\,(4\,a_1 + 24\,t + 120\,b_2)^4(-16\,t - 160\,b_2) +
450\,(4\,a_1 + 24\,t + 120\,b_2)(-3 + 12\,(-8\,b_1 - 80\,a_2)^2 - 4\,(-16\,t - 160\,b_2)^2) + 14400\,t + 259200\,b_2 + 1800\,(4\,a_1 +
24\,t + 120\,b_2)^2(-16\,t - 224\,b_2))(2\,x + 12\,b_1 + 60\,a_2)^4 + (-480\,(4\,a_1 + 24\,t + 120\,b_2)^5(-8\,b_1 - 80\,a_2) +
14400\,(4\,a_1 + 24\,t + 120\,b_2)^2(-8\,b_1 - 80\,a_2)(-16\,t - 160\,b_2) + 7200\,(4\,a_1 + 24\,t + 120\,b_2)(-8\,b_1 - 48\,a_2) -
2400\,(4\,a_1 + 24\,t + 120\,b_2)^3(-16\,b_1 - 128\,a_2) - 14400\,(-8\,b_1 - 80\,a_2)(-16\,t - 160\,b_2) - 460800\,(-16\,t -
160\,b_2)a_2 + 921600\,(-8\,b_1 - 80\,a_2)b_2)(2\,x + 12\,b_1 + 60\,a_2)^3 + (1710\,(4\,a_1 + 24\,t + 120\,b_2)^5 - 60\,(4\,a_1 + 24\,t +
120\,b_2)^7 + 5\,(4\,a_1 + 24\,t + 120\,b_2)^9 - 900\,(4\,a_1 + 24\,t + 120\,b_2)^3(7 + 4\,(-8\,b_1 - 80\,a_2)^2 - 12\,(-16\,t -
160\,b_2)^2) + 675\,(4\,a_1 + 24\,t + 120\,b_2)(7 + 16\,(-8\,b_1 - 80\,a_2)^2 + 16\,(-16\,t - 160\,b_2)^2) + 345600\,t +
4492800\,b_2 - 21600\,(-8\,b_1 - 80\,a_2)^2(-16\,t - 160\,b_2) - 21600\,(-16\,t - 160\,b_2)^3 + 691200\,(4\,a_1 + 24\,t +
120\,b_2)^2b_2 - 1800\,(4\,a_1 + 24\,t + 120\,b_2)^4(-64\,t - 448\,b_2))(2\,x + 12\,b_1 + 60\,a_2)^2 + (-240\,(4\,a_1 + 24\,t +
120\,b_2)^7(-8\,b_1 - 80\,a_2) - 7200\,(4\,a_1 + 24\,t + 120\,b_2)^4(-8\,b_1 - 80\,a_2)(-16\,t - 160\,b_2) + 10800\,(4\,a_1 + 24\,t +
120\,b_2)(-24\,b_1 - 400\,a_2 + 4\,(-8\,b_1 - 80\,a_2)^3 + 4\,(-8\,b_1 - 80\,a_2)(-16\,t - 160\,b_2)^2) + 3600\,(4\,a_1 + 24\,t +
120\,b_2)^3(-24\,b_1 - 176\,a_2) + 720\,(4\,a_1 + 24\,t + 120\,b_2)^5(-56\,b_1 - 400\,a_2) + 21600\,(-8\,b_1 - 80\,a_2)(-16\,t -
160\,b_2) + 1382400\,(-16\,t - 160\,b_2)a_2 - 2764800\,(-8\,b_1 - 80\,a_2)b_2 - 43200\,(4\,a_1 + 24\,t + 120\,b_2)^2((-8\,b_1 -
80\,a_2)(-16\,t - 160\,b_2) + 32\,(-16\,t - 160\,b_2)a_2 - 64\,(-8\,b_1 - 80\,a_2)b_2))(2\,x + 12\,b_1 + 60\,a_2) - 90\,(4\,a_1 +
24\,t + 120\,b_2)^8(-16\,t - 160\,b_2) + 90\,(4\,a_1 + 24\,t + 120\,b_2)^5(-107 + 28\,(-8\,b_1 - 80\,a_2)^2 + 12\,(-16\,t -$$

$$160\,b_2)^2)+5400\,(4\,a_1+24\,t+120\,b_2)^2(-176\,t-2464\,b_2+4\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)+4\,(-16\,t-160\,b_2)^3)-120\,(4\,a_1+24\,t+120\,b_2)^6(-80\,t-1248\,b_2)+900\,(4\,a_1+24\,t+120\,b_2)^4(-464\,t-4000\,b_2)-225\,(4\,a_1+24\,t+120\,b_2)^3(11+80\,(-8\,b_1-80\,a_2)^2+80\,(-16\,t-160\,b_2)^2+4096\,(-8\,b_1-80\,a_2)a_2+8192\,(-16\,t-160\,b_2)b_2)-675\,(4\,a_1+24\,t+120\,b_2)(-7+56\,(-8\,b_1-80\,a_2)^2+88\,(-16\,t-160\,b_2)^2-4096\,(-8\,b_1-80\,a_2)a_2-131072\,{a_2}^2-524288\,{b_2}^2)+5529600\,(-8\,b_1-80\,a_2)^2b_2-5529600\,(-16\,t-160\,b_2)^2b_2+1857600\,b_2)$$

and

$$d(x,t) = 2024+518400\,(-16\,t-160\,b_2)^4+356400\,(-8\,b_1-80\,a_2)^2+874800\,(-16\,t-160\,b_2)^2+3720\,(4\,a_1+24\,t+120\,b_2)^8+120\,(4\,a_1+24\,t+120\,b_2)^{10}+530841600\,{a_2}^2+2123366400\,{b_2}^2+(1+(2\,x+12\,b_1+60\,a_2)^2+(4\,a_1+24\,t+120\,b_2)^2)^6+518400\,(-8\,b_1-80\,a_2)^4+(-1440\,(4\,a_1+24\,t+120\,b_2)^4+720\,(4\,a_1+24\,t+120\,b_2)^5(-16\,t-160\,b_2)+240\,(4\,a_1+24\,t+120\,b_2)^2(56+135\,(-8\,b_1-80\,a_2)^2-45\,(-16\,t-160\,b_2)^2)+32400\,(4\,a_1+24\,t+120\,b_2)(-16\,t-288\,b_2)+7200\,(4\,a_1+24\,t+120\,b_2)^3(-48\,t-544\,b_2)+3360+32400\,(-8\,b_1-80\,a_2)^2-54000\,(-16\,t-160\,b_2)^2+2764800\,(-8\,b_1-80\,a_2)a_2+5529600\,(-16\,t-160\,b_2)b_2)(2\,x+12\,b_1+60\,a_2)^4+(-960\,(4\,a_1+24\,t+120\,b_2)^6(-8\,b_1-80\,a_2)+57600\,(4\,a_1+24\,t+120\,b_2)^3(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)-43200\,(4\,a_1+24\,t+120\,b_2)^2(-24\,b_1-272\,a_2)-7200\,(4\,a_1+24\,t+120\,b_2)^4(-48\,b_1-448\,a_2)-345600\,b_1-5529600\,a_2-86400\,(-8\,b_1-80\,a_2)^3-86400\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)^2+172800\,(4\,a_1+24\,t+120\,b_2)((-8\,b_1-80\,a_2)(-16\,t-160\,b_2)-32\,(-16\,t-160\,b_2)a_2+64\,(-8\,b_1-80\,a_2)b_2))(2\,x+12\,b_1+60\,a_2)^3+(13440\,(4\,a_1+24\,t+120\,b_2)^6+240\,(4\,a_1+24\,t+120\,b_2)^8-240\,(4\,a_1+24\,t+120\,b_2)^4(-326+45\,(-8\,b_1-80\,a_2)^2-135\,(-16\,t-160\,b_2)^2)+480\,(4\,a_1+24\,t+120\,b_2)^2(-76+135\,(-8\,b_1-80\,a_2)^2+1215\,(-16\,t-160\,b_2)^2)-129600\,(4\,a_1+24\,t+120\,b_2)^3(-32\,t-256\,b_2)-12960\,(4\,a_1+24\,t+120\,b_2)^5(-32\,t-256\,b_2)-64800\,(4\,a_1+24\,t+120\,b_2)(96\,t+1280\,b_2+4\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)+4\,(-16\,t-160\,b_2)^3)+12144-97200\,(-8\,b_1-80\,a_2)^2+32400\,(-16\,t-160\,b_2)^2+530841600\,{a_2}^2-33177600\,(-16\,t-160\,b_2)b_2+2123366400\,{b_2}^2)(2\,x+12\,b_1+60\,a_2)^2+(-360\,(4\,a_1+24\,t+120\,b_2)^8(-8\,b_1-80\,a_2)-17280\,(4\,a_1+24\,t+120\,b_2)^5(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)-1440\,(4\,a_1+24\,t+120\,b_2)^6(-8\,b_1-240\,a_2)+32400\,(4\,a_1+24\,t+120\,b_2)^4(-8\,b_1+112\,a_2)+64800\,(4\,a_1+24\,t+120\,b_2)^2(40\,b_1+752\,a_2+4\,(-8\,b_1-80\,a_2)^3+4\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)^2)-777600\,(4\,a_1+24\,t+120\,b_2)((-8\,b_1-80\,a_2)(-16\,t-160\,b_2)+64\,(-16\,t-160\,b_2)a_2-128\,(-8\,b_1-80\,a_2)b_2)-172800\,(4\,a_1+24\,t+120\,b_2)^3(3\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)+32\,(-16\,t-160\,b_2)a_2-64\,(-8\,b_1-80\,a_2)b_2)+648000\,b_1+8553600\,a_2+259200\,(-8\,b_1-80\,a_2)^3+1296000\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)^2-33177600\,(-8\,b_1-80\,a_2)^2a_2+33177600\,(-16\,t-160\,b_2)^2a_2-132710400\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)b_2)(2\,x+12\,b_1+60\,a_2)+120\,(-8\,b_1-80\,a_2)(2\,x+12\,b_1+60\,a_2)^9-120\,(4\,a_1+24\,t+120\,b_2)^9(-16\,t-160\,b_2)+80\,(4\,a_1+24\,t+120\,b_2)^6(191+63\,(-8\,b_1-80\,a_2)^2+27\,(-16\,t-160\,b_2)^2)-2160\,(4\,a_1+24\,t+120\,b_2)^5(-240\,t-4576\,b_2)+21600\,(4\,a_1+24\,t+120\,b_2)^3(368\,t+3488\,b_2+4\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)+4\,(-16\,t-160\,b_2)^3)-1440\,(4\,a_1+24\,t+120\,b_2)^7(-80\,t-864\,b_2)+240\,(4\,a_1+24\,t+120\,b_2)^4(599+135\,(-8\,b_1-80\,a_2)^2-225\,(-16\,t-160\,b_2)^2-11520\,(-8\,b_1-80\,a_2)a_2-23040\,(-16\,t-160\,b_2)b_2)-16200\,(4\,a_1+24\,t+120\,b_2)(-496\,t-6240\,b_2+80\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)+16\,(-16\,t-160\,b_2)^3+4096\,(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)a_2-4096\,(-8\,b_1-80\,a_2)^2b_2+4096\,(-16\,t-160\,b_2)^2b_2)+24\,(4\,a_1+24\,t+120\,b_2)^2(3881+12150\,(-8\,b_1-80\,a_2)^2+28350\,(-16\,t-160\,b_2)^2+691200\,(-8\,b_1-80\,a_2)a_2+22118400\,{a_2}^2+88473600\,{b_2}^2)+1036800\,(-8\,b_1-80\,a_2)^2(-16\,t-160\,b_2)^2+46080\,a_2(2\,x+12\,b_1+60\,a_2)^7-24883200\,(-8\,b_1-80\,a_2)a_2-82944000\,(-16\,t-160\,b_2)b_2+(-120\,(4\,a_1+24\,t+120\,b_2)^2+360\,(4\,a_1+24\,t+120\,b_2)(-16\,t-160\,b_2)+120)(2\,x+12\,b_1+60\,a_2)^8+(480\,(4\,a_1+24\,t+120\,b_2)^2-240\,(4\,a_1+24\,t+120\,b_2)^4+960\,(4\,a_1+24\,t+120\,b_2)^3(-16\,t-160\,b_2)+2320+2160\,(-8\,b_1-80\,a_2)^2+5040\,(-16\,t-160\,b_2)^2-1440\,(4\,a_1+24\,t+120\,b_2)(-64\,t-960\,b_2))(2\,x+12\,b_1+60\,a_2)^6+(-720\,(4\,a_1+24\,t+120\,b_2)^4(-8\,b_1-80\,a_2)-17280\,(4\,a_1+24\,t+120\,b_2)(-8\,b_1-80\,a_2)(-16\,t-160\,b_2)+4320\,(4\,a_1+24\,t+120\,b_2)^2(-8\,b_1-176\,a_2)+51840\,b_1+103680\,a_2)(2\,x+12\,b_1+60\,a_2)^5$$

is a solution to the Lakshmanan Porsezian Daniel equation (1.1).

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