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Discrete Adomian Decomposition Method for Fuzzy Convection-Diffusion Equation

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Abstract

In this paper, the discrete Adomian decomposition method (DADM) is applied to obtain the approximate solution of fuzzy convection-diffusion equation (FCDE). The numerical results are compared with the exact solution. It is shown that this method is accurate and effective for FCDE. Also, the analytical-approximate solution of this equation by Adomian decomposition method (ADM) is offered.

Keywords: Discrete Adomian decomposition method; Fuzzy differential equation; convection-diffusion equation; analytical-approximate methods.

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1 Introduction

The Adomian decomposition method is a powerful method to solve the linear or nonlinear differential or integral equations [1], [2]. The discrete Adomian decomposition method was first proposed by Bratsos et al.[3] applied to discrete nonlinear Schrodinger equations. Zhu et al. [4] have developed the DADM to 2D Burgers difference equations, Abdulghafor M. Al-Rozbayani et al.[5] applied this method to nonlinear difference scheme of generalized Burgers-Huxley equation. In this work, we apply this method for fuzzy convection-diffusion equation and obtain the numerical solution of this equation. Also the analytical-approximate solution of FCDE by Adomian decomposition method is offered.

Convection diffusion equation (CDE) is a combination of the diffusion and convection equations, and describes physical phenomena where particles, energy, or other physical quantities are transferred inside a physical system due to two processes. This equation is solved by many methods such as finite difference method and Alternating Group Iterative Method [6], [7].

In recent years, some numerical and analytical methods were proposed in order to solve fuzzy differential equations such as [8], [9], [10], [11], [12], [13], [14], [15], [16]. In this work, we consider the following fuzzy case of convection-diffusion equation and apply the ADM and DADM to solve it.

$$\frac{\partial \widetilde{u}}{\partial t} + \alpha \frac{\partial \widetilde{u}}{\partial x} = \gamma \frac{\partial^2 \widetilde{u}}{\partial x^2}, \quad 0 \le x \le l, t \ge 0, \tag{1.1}$$

with the initial condition,

$$\widetilde{u}(x,0) = \widetilde{f}(t), \quad 0 \le x \le l, \tag{1.2}$$

where $\widetilde{u}(x,t)$ is unknown fuzzy function, $\widetilde{f}(x)$ is known fuzzy function, and α, γ are known crisp constants.

In section 2, we treat some fuzzy concepts briefly, then in section 3, we apply the ADM for FCDE, in section 4, we present the DADM for FCDE, and in section 5, we solve two examples and offer the analytical- approximate and numerical solutions of them by ADM and DADM respectively.

2 Preliminaries

In this section, we recall some basic definitions of fuzzy sets theory mentioned in [17], [18], [19], [20], [21], [22], [23].

Definition 2.1. A fuzzy parametric number u is a pair $(\underline{u}(r), \overline{u}(r))$, $0 \le r \le 1$, which satisfy the following requirements :

1. $\underline{u}(r)$ is a bounded left continuous non-decreasing function over $[0, 1]$.
2. $\overline{u}(r)$ is a bounded left continuous non-increasing function over $[0, 1]$.
3. $\underline{u}(r) \le \overline{u}(r),\ 0 \le r \le 1$.

The set of all these fuzzy numbers is denoted by $\mathbb{E}^1$. For $u = (\underline{u}, \overline{u}), v = (\underline{v}, \overline{v}) \in \mathbb{E}^1$, $k \in \mathbb{R}$ the addition, multiplication and the scaler multiplication of fuzzy numbers are defined by

$\underline{(u + v)}(r) = \underline{u}(r) + \underline{v}(r)$,
$\overline{(u + v)}(r) = \overline{u}(r) + \overline{v}(r)$,

$(\underline{u.v})(r) =$
$\min\{\underline{u}(r).\underline{v}(r), \underline{u}(r).\overline{v}(r), \overline{u}(r).\underline{v}(r), \overline{u}(r).\overline{v}(r)\},$
$(\overline{u.v})(r) =$
$\max\{\underline{u}(r).\underline{v}(r), \underline{u}(r).\overline{v}(r), \overline{u}(r).\underline{v}(r), \overline{u}(r).\overline{v}(r)\},$
$\underline{ku}(r) = k\underline{u}(r), \quad \overline{ku}(r) = k\overline{u}(r), \quad k \geq 0,$
$\underline{ku}(r) = k\overline{u}(r), \quad \overline{ku}(r) = k\underline{u}(r), \quad k \leq 0.$

Definition 2.2. For arbitrary fuzzy numbers $\widetilde{u} = (\underline{u}, \overline{u}), \widetilde{v} = (\underline{v}, \overline{v})$ the quantity

$$D(\widetilde{u}, \widetilde{v}) = \sup_{0 \leq r \leq 1} \{\max[|\underline{u}(r) - \underline{v}(r)|, |\overline{u}(r) - \overline{v}(r)|]\}$$

is the Hausdorff distance between $\widetilde{u}$ and $\widetilde{v}$.

It is shown that $\mathbb{E}^1, D$ is a complete metric space [21].

Definition 2.3. A function $f : \mathbb{R}^1 \longrightarrow \mathbb{E}^1$ is called a fuzzy function. If for arbitrary fixed $t_0 \in \mathbb{E}^1$ and $\varepsilon > 0$ such that, $|t - t_0| < \delta \Longrightarrow D(f(t), f(t_0)) < \varepsilon$ exists, f is said to be continuous.

Definition 2.4. Let $u, v \in \mathbb{E}^1$. If there exists $w \in \mathbb{E}^1$ such that $u = v + w$, then w is called the H-difference of u, v and it is denoted $u \ominus v$.

Definition 2.5. Let $a, b \in \mathbb{R}$ and $f : (a, b) \to \mathbb{E}^1$. Fix $t_0 \in (a, b)$. We say F is strongly generalized differentiable at t_0, if there exists $f'(t_0) \in \mathbb{E}^1$ such that

(i) for all $h > 0$ sufficiently close to 0, there exist $f(t_0 + h) \ominus f(t_0), f(t_0) \ominus f(t_0 - h)$ and the limits

$$\lim_{h \to 0^+} \frac{f(t_0 + h) \ominus f(t_0)}{h} = \lim_{h \to 0^+} \frac{f(t_0) \ominus f(t_0 - h)}{h} = f'(t_0),$$

or

(ii) for all $h > 0$ sufficiently close to 0, there exist $f(t_0 - h) \ominus f(t_0), f(t_0) \ominus f(t_0 + h)$ and the limits

$$\lim_{h \to 0^+} \frac{f(t_0 - h) \ominus f(t_0)}{-h} = \lim_{h \to 0^+} \frac{f(t_0) \ominus f(t_0 + h)}{-h} = f'(t_0),$$

or

(iii) for all $h > 0$ sufficiently close to 0, there exist $f(t_0 + h) \ominus f(t_0), f(t_0 - h) \ominus f(t_0)$ and the limits

$$\lim_{h \to 0^+} \frac{f(t_0 + h) \ominus f(t_0)}{h} = \lim_{h \to 0^+} \frac{f(t_0 - h) \ominus f(t_0)}{-h} = f'(t_0),$$

or

(iv) for all $h > 0$ sufficiently close to 0, there exist $f(t_0) \ominus f(t_0 + h), f(t_0) \ominus f(t_0 - h)$ and the limits

$$\lim_{h \to 0^+} \frac{f(t_0) \ominus f(t_0 + h)}{-h} = \lim_{h \to 0^+} \frac{f(t_0) \ominus f(t_0 - h)}{h} = f'(t_0).$$

(h and $(-h)$ at denominators mean $\frac{1}{h}.$ and $-\frac{1}{h}.$ respectively)[17].

Theorem 2.1. *Let $f : (a, b) \to \mathbb{E}^1$ be strongly generalized differentiable on each point $t \in (a, b)$ in the sense of Definition 2.5, (3) or (4). Then $f'(x) \in \mathbb{R}$ for all $t \in (a, b)$ (see[17]).*

Theorem 2.2. *Let $f : \mathbb{R}^1 \longrightarrow \mathbb{E}^1$ be a function and denote $f(t) = (\underline{f}(t, r), \overline{f}(t, r))$, for each $r \in [0, 1]$. Then*

(1) If f is differentiable in the first form (i), then $\underline{f}(t, r)$ and $\overline{f}(t, r)$ are differentiable functions and $f'(t) = (\underline{f'}(t, r), \overline{f'}(t, r))$,

(2) If f is differentiable in the second form (ii), then $\underline{f}(t, r)$ and $\overline{f}(t, r)$ are differentiable functions and $f'(t) = (\overline{f'}(t, r), \underline{f'}(t, r))$ (see[18]).

Definition 2.6. Let $a, b \in \mathbb{R}$ and $f : (a, b) \to \mathbb{E}^1$ and $t_0 \in (a, b)$. We define the n-th order differential of f as follows: We say that f is strongly generalized differentiable of n-th order at t_0, if there exists an element $f^{(s)}(t_0) \in \mathbb{E}^1 \quad \forall s = 1, \dots, n$ such that

(i) for all $h > 0$ sufficiently close to 0, there exist $f^{(s-1)}(t_0 + h) \ominus f^{(s-1)}(t_0), f^{(s-1)}(t_0) \ominus f^{(s-1)}(t_0 - h)$ and the limits

$$\lim_{h \to 0^+} \frac{f^{(s-1)}(t_0 + h) \ominus f^{(s-1)}(t_0)}{h} = \lim_{h \to 0^+} \frac{f^{(s-1)}(t_0) \ominus f^{(s-1)}(t_0 - h)}{h} = f^{(s)}(t_0),$$

or

(ii) for all $h > 0$ sufficiently close to 0, there exist $f^{(s-1)}(t_0 - h) \ominus f^{(s-1)}(t_0), f^{(s-1)}(t_0) \ominus f^{(s-1)}(t_0 + h)$ and the limits

$$\lim_{h \to 0^+} \frac{f^{(s-1)}(t_0 - h) \ominus f^{(s-1)}(t_0)}{-h} = \lim_{h \to 0^+} \frac{f^{(s-1)}(t_0) \ominus f^{(s-1)}(t_0 + h)}{-h} = f^{(s)}(t_0),$$

or

(iii) for all $h > 0$ sufficiently close to 0, there exist $f^{(s-1)}(t_0 + h) \ominus f^{(s-1)}(t_0), f^{(s-1)}(t_0 - h) \ominus f^{(s-1)}(t_0)$ and the limits

$$\lim_{h \to 0^+} \frac{f^{(s-1)}(t_0 + h) \ominus f^{(s-1)}(t_0)}{h} = \lim_{h \to 0^+} \frac{f^{(s-1)}(t_0 - h) \ominus f^{(s-1)}(t_0)}{-h} = f^{(s)}(t_0),$$

or

(iv) for all $h > 0$ sufficiently close to 0, there exist $f^{(s-1)}(t_0) \ominus f^{(s-1)}(t_0 + h), f^{(s-1)}(t_0) \ominus f^{(s-1)}(t_0 - h)$ and the limits

$$\lim_{h \to 0^+} \frac{f^{(s-1)}(t_0) \ominus f^{(s-1)}(t_0 + h)}{-h} = \lim_{h \to 0^+} \frac{f^{(s-1)}(t_0) \ominus f^{(s-1)}(t_0 - h)}{h} = f^{(s)}(t_0).$$

(h and $(-h)$ at denominators mean $\frac{1}{h}$. and $-\frac{1}{h}$. respectively $\forall s = 1, \dots, n$).

Remark 2.1. Note that by the above definition a fuzzy function is i-differentiable or ii-differentiable of order n if $f^{(s)}$ for $s = 1, \dots, n$ is i-differentiable or ii-differentiable. It is possible that the different orders have different kind i or ii differentiability.

For a given fuzzy function f ,we have two possibilities according to the definition 2.5 to obtain the derivative of f at t: $D_1(f(t)), D_2(f(t))$.

Then for each of these two derivatives, we have again two possibilities:

$$D_1(D_1(f(t)) = D^2_{1,1}(f(t))\ ,\ D_2(D_1(f(t)) = D^2_{2,1}(f(t)$$

and

$$D_1(D_2(f(t)) = D^2_{1,2}(f(t))\ ,\ D_2(D_2(f(t)) = D^2_{2,2}(f(t).$$

In similar we consider the n-order differential of f. For example

$$D^3_{1,2,1}(f(t)) = D_1(D_2(D_1(f(t)))).$$

3 The Adomian Decomposition Method for FCDE

In this case, we apply ADM to solve the Eq.(1.1)[2]. According to the definition 2.1 with assumption that $\widetilde{u}$ is i-differentiable in terms of x, t, we rewrite the Eq.(1.1) in the following form,

$$(\underline{u}_t, \overline{u}_t) + \alpha(\underline{u}_x, \overline{u}_x) = \gamma(\underline{u}_{xx}, \overline{u}_{xx}), \tag{3.1}$$

with the initial condition,

$$\widetilde{u}(x, 0) = (\underline{f}(x), \overline{f}(t)). \tag{3.2}$$

We consider the following cases, by attention to the signs of α and γ:

1. If α and $\gamma > 0$,

$$\begin{cases} \underline{u}_t + \alpha\underline{u}_x = \gamma\underline{u}_{xx} \\ \overline{u}_t + \alpha\overline{u}_x = \gamma\overline{u}_{xx} \end{cases} \tag{3.3}$$

2. If $\alpha > 0$ and $\gamma < 0$,

$$\begin{cases} \underline{u}_t + \alpha\underline{u}_x = \gamma\overline{u}_{xx} \\ \overline{u}_t + \alpha\overline{u}_x = \gamma\underline{u}_{xx} \end{cases} \tag{3.4}$$

3. If $\alpha < 0$ and $\gamma > 0$,

$$\begin{cases} \underline{u}_t + \alpha\overline{u}_x = \gamma\underline{u}_{xx} \\ \overline{u}_t + \alpha\underline{u}_x = \gamma\overline{u}_{xx} \end{cases} \tag{3.5}$$

4. If α and $\gamma < 0$,

$$\begin{cases} \underline{u}_t + \alpha\overline{u}_x = \gamma\overline{u}_{xx} \\ \overline{u}_t + \alpha\underline{u}_x = \gamma\underline{u}_{xx}. \end{cases} \tag{3.6}$$

And initial conditions,

$$\underline{u}(x, 0) = \underline{f}(x), \quad \overline{u}(x, 0) = \overline{f}(x). \tag{3.7}$$

According to the description of the ADM, we consider $\underline{u} = \sum_{m=0}^{+\infty} \underline{u}_m$ and $\overline{u} = \sum_{m=0}^{+\infty} \overline{u}_m$. Then, we solve the given systems of partial differential equations (3.3)-(3.6).
Hence, we consider,

$$\underline{u}_0 = \underline{f}, \quad \overline{u}_0 = \overline{f}. \tag{3.8}$$

For $m \geq 1$,
in case.1:

$$\underline{u}_m = \int_0^t (-\alpha\frac{\partial \underline{u}_{m-1}}{\partial x} + \gamma\frac{\partial^2 \underline{u}_{m-1}}{\partial x^2})d\tau, \qquad \overline{u}_m = \int_0^t (-\alpha\frac{\partial \overline{u}_{m-1}}{\partial x} + \gamma\frac{\partial^2 \overline{u}_{m-1}}{\partial x^2})d\tau, \tag{3.9}$$

in case.2:

$$\underline{u}_m = \int_0^t (-\alpha\frac{\partial \underline{u}_{m-1}}{\partial x} + \gamma\frac{\partial^2 \overline{u}_{m-1}}{\partial x^2})d\tau, \qquad \overline{u}_m = \int_0^t (-\alpha\frac{\partial \overline{u}_{m-1}}{\partial x} + \gamma\frac{\partial^2 \underline{u}_{m-1}}{\partial x^2})d\tau, \tag{3.10}$$

in case.3:

$$\underline{u}_m = \int_0^t (-\alpha \frac{\partial \overline{u}_{m-1}}{\partial x} + \gamma \frac{\partial^2 \underline{u}_{m-1}}{\partial x^2}) d\tau, \qquad \overline{u}_m = \int_0^t (-\alpha \frac{\partial \underline{u}_{m-1}}{\partial x} + \gamma \frac{\partial^2 \overline{u}_{m-1}}{\partial x^2}) d\tau, \tag{3.11}$$

in case.4:

$$\underline{u}_m = \int_0^t (-\alpha \frac{\partial \overline{u}_{m-1}}{\partial x} + \gamma \frac{\partial^2 \overline{u}_{m-1}}{\partial x^2}) d\tau, \qquad \overline{u}_m = \int_0^t (-\alpha \frac{\partial \underline{u}_{m-1}}{\partial x} + \gamma \frac{\partial^2 \underline{u}_{m-1}}{\partial x^2}) d\tau. \tag{3.12}$$

In other case of differentiability of $\widetilde{u}$ in terms of x or t, we can construct other four cases, similar to the (3.9)-(3.12).

4 The Discrete Adomian Decomposition Method for FCDE

To apply the DADM to Eq.(1.1), we denote the discrete approximation of $u(x,t)$ at the grid point (ih, nk) by u_i^n $(i = 0, 1, 2, \ldots, N; n = 0, 1, 2, \ldots)$, where $h = \frac{l}{N}$ is the special step size and k represent time increment [5], [3].

We can rewrite the discrete operator form of u_t, u_x, u_{xx} in the form of $D_k^+ u_i^n, D_h u_i^n, D_h^2 u_i^n$ respectively, where that

$$D_k^+ u_i^n = \frac{u_i^{n+1} - u_i^n}{k}, \quad D_h u_i^n = \frac{u_{i+1}^n - u_{i-1}^n}{2h}, \quad D_h^2 u_i^n = \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{h^2}.$$

The inverse discrete operator $(D_k^+)^{-1}$ is given by,

$$(D_k^+)^{-1} u_i^n = k \sum_{m=0}^{n-1} u_i^m. \tag{4.1}$$

Thus $(D_k^+)^{-1} D_k^+ u_i^n = u_i^n - u_i^0$.

We consider $\underline{u}_i^n = \sum_{m=0}^{+\infty} \underline{u}_{i,m}^n$ and $\overline{u}_i^n = \sum_{m=0}^{+\infty} \overline{u}_{i,m}^n$. By rewriting the discrete operator form of the system of equations (3.9), (3.10), (3.11) and (3.12), we obtain four new systems by initial conditions $\underline{u}_i^0 = \underline{f}_i$ and $\overline{u}_i^0 = \overline{f}_i$, where $f_i = f(ih)$.

Then by applying the inverse operator $(D_k^+)^{-1}$ to the discrete operator form of the system of equations (3.9), (3.10), (3.11) and (3.12), we construct the following relations to obtain the discrete solution of the Eq.(1.1).

At first, we consider,

$$\underline{u}_{i,0}^n = \underline{f}_0, \qquad \overline{u}_{i,0}^n = \overline{f}_0. \tag{4.2}$$

For $m \geq 1$,

in case.1:

$$\underline{u}_{i,m}^n = (D_k^+)^{-1}(-\alpha D_h \underline{u}_{i,m-1}^n + \gamma D_h^2 \underline{u}_{i,m-1}^n), \qquad \overline{u}_{i,m}^n = (D_k^+)^{-1}(-\alpha D_h \overline{u}_{i,m-1}^n + \gamma D_h^2 \overline{u}_{i,m-1}^n), \tag{4.3}$$

in case.2:

$$\underline{u}_{i,m}^n = (D_k^+)^{-1}(-\alpha D_h \underline{u}_{i,m-1}^n + \gamma D_h^2 \overline{u}_{i,m-1}^n), \qquad \overline{u}_{i,m}^n = (D_k^+)^{-1}(-\alpha D_h \overline{u}_{i,m-1}^n + \gamma D_h^2 \underline{u}_{i,m-1}^n), \tag{4.4}$$

in case.3:

$$\underline{u}^n_{i,m} = (D^+_k)^{-1}(-\alpha D_h \overline{u}^n_{i,m-1} + \gamma D^2_h \underline{u}^n_{i,m-1}), \qquad \overline{u}^n_{i,m} = (D^+_k)^{-1}(-\alpha D_h \underline{u}^n_{i,m-1} + \gamma D^2_h \overline{u}^n_{i,m-1}), \tag{4.5}$$

in case.4:

$$\underline{u}^n_{i,m} = (D^+_k)^{-1}(-\alpha D_h \overline{u}^n_{i,m-1} + \gamma D^2_h \overline{u}^n_{i,m-1}), \qquad \overline{u}^n_{i,m} = (D^+_k)^{-1}(-\alpha D_h \underline{u}^n_{i,m-1} + \gamma D^2_h \underline{u}^n_{i,m-1}). \tag{4.6}$$

Also in other cases of differentiability of $\widetilde{u}$ in terms of x or t, we can construct other four cases, similar to the (4.3)-(4.6).

5 Numerical Examples

In this case, we solve two sample fuzzy convection-diffusion equations by ADM and DADM.

Example 5.1. *We consider Eq.(1.1) with $\alpha = 0.0001, \gamma = 1.0001$ and $\widetilde{f}(x) = ((2r^2-1)e^x, (2-r)e^x)$, also we suppose $\widetilde{u}(x,t)$ is i-differentiable in terms of x, t, and consider Eq.(3.9). By applying ADM and choosing $\widetilde{u}_0 = ((2r^2-1)e^x, (2-r)e^x)$, we have,*

$$\begin{aligned}\widetilde{u}_1 &= ((2r^2-1)e^x t, (2-r)e^x t),\\ \widetilde{u}_2 &= ((2r^2-1)e^x(\frac{1}{2}t^2), (2-r)e^x(\frac{1}{2}t^2)),\\ \widetilde{u}_3 &= ((2r^2-1)e^x(\frac{1}{6}t^3), (2-r)e^x(\frac{1}{6}t^3)),\\ &\ddots\end{aligned}$$

In general $\widetilde{u} = \widetilde{u}_0 + \widetilde{u}_1 + \widetilde{u}_2 + \cdots$, that it converges to the exact solution $\widetilde{u} = ((2r^2-1)e^{x+t}, (2-r)e^{x+t})$.

Now, we apply DADM, by choosing $\widetilde{u}^n_{i,0} = ((2r^2-1)e^{ih}, (2-r)e^{ih})$, $h = 0.1, k = 0.001$ and Eq.(4.3). The results are shown in table 1 with 5 iterations.

Table 1

t	x	r	$\underline{u}_{DADM}$	$\underline{u}_{exact}$	$\overline{u}_{DADM}$	$\overline{u}_{exact}$
		1	0.27185	0.27182	0.2785	0.27182
0.5	0.5	$\frac{1}{2}$	-1.35928	-1.35914	4.07784	4.07742
		$\frac{1}{4}$	-2.37874	-2.37849	4.75748	4.75699

Example 5.2. *In this example we consider Eq.(1.1) with $\alpha = 0, \gamma = -1$ and $\widetilde{f}(x) = ((4r-3)e^x, (2-r^2)e^x)$, also we suppose $\widetilde{u}(x,t)$ is ii-differentiable in terms of t and i-differentiable in terms of x. Therefore,*

$$\begin{aligned}\overline{u}_t &= -\overline{u}_{xx}\\ \underline{u}_t &= -\underline{u}_{xx}.\end{aligned} \tag{5.1}$$

By applying ADM and DADM, we construct following formulas,

$$\begin{aligned}\overline{u}_m &= \textstyle\int_0^t(-\gamma\frac{\partial^2 \overline{u}_{m-1}}{\partial x^2})d\tau, & \underline{u}_m &= \textstyle\int_0^t(-\gamma\frac{\partial^2 \underline{u}_{m-1}}{\partial x^2})d\tau,\\ \overline{u}^n_{i,m} &= (D^+_k)^{-1}(-\gamma D^2_h \overline{u}^n_{i,m-1}), & \underline{u}^n_{i,m} &= (D^+_k)^{-1}(-\gamma D^2_h \underline{u}^n_{i,m-1}).\end{aligned} \tag{5.2}$$

Therefore, by choosing $\widetilde{u}_0 = ((4r-3)e^x, (2-r^2)e^x)$, the ADM gives us the following results,

$$\underline{u}_1 = (4r-3)e^x(-t), \quad \overline{u}_2 = (2-r^2)e^x(-t),$$
$$\underline{u}_2 = (4r-3)e^x(\frac{1}{2}t^2), \quad \overline{u}_2 = (2-r^2)e^x(\frac{1}{2}t^2),$$
$$\underline{u}_3 = (4r-3)e^x(-\frac{1}{6}t^3), \quad \overline{u}_3 = (2-r^2)e^x(-\frac{1}{6}t^3),$$
$$\vdots$$

In general $\underline{u} = \underline{u}_0 + \underline{u}_1 + \underline{u}_2 + \cdots$, *and* $\overline{u} = \overline{u}_0 + \overline{u}_1 + \overline{u}_2 + \cdots$. *That it converges to the exact solution* $\widetilde{u} = ((4r-3)e^{x-t}, (2-r^2)e^{x-t})$.

Now, by applying DADM and choosing $\widetilde{u}_{i,0}^n = ((4r-3)e^{ih}, (2-r^2)e^{ih})$ *and also* $h = 0.1, k = 0.001$. *The results are shown in table 2 with* 5 *iterations.*

Table 2

t	x	r	$\underline{u}_{DADM}$	$\underline{u}_{exact}$	$\overline{u}_{DADM}$	$\overline{u}_{exact}$
		1	1.3493	1.3498	1.3493	1.3498
0.3	0.6	$\frac{2}{3}$	-0.4498	-0.4499	2.0989	2.0998
		$\frac{1}{4}$	-2.6986	-2.6997	2.6143	2.6153

6 Conclusion

In this work, we presented the discrete Adomian decomposition method and applied the Adomian decomposition method to obtain the numerical and analytical-approximate solutions of fuzzy convection-diffusion equation, and we compared the results with the exact solutions to show the efficiency of these methods.

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Competing Interests

The authors declare that no competing interests exist.

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