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Comparative study of reliability parameter of a system under different types of distribution functions

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In this paper a two unit standby system with single repair facility has been considered. When a working unit fails, it is immediately taken over by standby unit and repair on the failed unit is started immediately. Taking two types of distribution, namely, Weibull and Erlangian, various system effectiveness measures such as MTSF, Availability and Busy Periods are compared and results are interpreted numerically. Regenerative Point Technique and Semi-Markov process have been employed in this paper to find the results. Results are supported with numerical data also. Failure time distributions are taken to be exponential whereas the repair times are particular. The result obtained from this can be applied to study complex system where small change in the value of one variable affects the system measures to a great extent.

Key words: MTSF, availability, busy period, regenerative point technique, Semi-Markov process.

INTRODUCTION

Reliability measures of a component for a two-component system with repair facility were obtained by several authors under different assumptions. Harris (1968), considered a two-unit parallel redundant system in which failure times of the components are dependent and distributed as bivariate exponential of Marshall and Olkin (1967), to derive the mean time to system failure using the supplementary variable technique for an arbitrary repair time distribution. Osaki (1980) extended the analysis to obtain the availability of the system by using a variant of Semi-Markov process with non-regeneration point technique. Jye-Chyi (1992) studied the effects of dependence on modeling system reliability via multivariate Weibull distribution. In reliability theory, the steady-state availability of a repairable system is an important feature.

In this paper, we have taken two different types of distributions namely Weibull and Erlangian to study the reliability measures such as mean time to system failure, steady-state availability, busy period of the repairman in repairing the failed units and profit analysis and compare them. Very few authors have attempted to compare the

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system effectiveness measures before now. The comparison can be helpful in studying the performance of complex system from reliability point of view and will be fruitful to system managers for evaluating the profit analysis of working systems. In the present work a two-unit standby system with single repair facility is considered. As soon as the main unit fails, it is taken over by the standby unit and the failed unit is sent for repair. To improve reliability, the concept of preventive maintenance is also added. When both the units fail, the whole system is shut down to prevent any further losses and the system starts afresh.

Literature review

Earlier Gopalan and Waghmare (1992) worked on evaluating cost benefit analysis of single server n-unit system. Gupta (2002), Yong et al. (2012), Li and Zuo (2008) have calculated reliability measures of k out of n systems. Jane and Laih (2010) has provided dynamic algorithm for multistate two-terminal reliability. Whereas Lie et al. (1977) has provided calculation techniques for Availability. Marshall and Olkin (1967b), Nadarajah and Kotz (2005), Osaki (1970) provided excellent results on BVE Weibull and Markov Renewal Process which are the backbone of reliability literature. Paul and Chandrasekar (1997) have introduced the idea of dependent structure for failure and repair times. On the other hand, Pijnenburg et al. (1993) gave the idea of dependent parallel system. Rander et al. (1992) and Singh et al. (1986) discussed two unit cold standby concept considering various assumptions regarding failures, repairs, inspections and replacement. Yearout et al. (1986) provide excellent review on standby redundancy.

Assumptions used in the model

(a) The system consists of two main units along with an associate unit in which one main unit is kept on standby mode.
(b) Whenever an operational unit fails, it is immediately taken over by standby unit.
(c) There is a single repairman which repairs the failed unit on priority basis.
(d) If both the main units fail the system shuts down.
(e) After repair all units work as new.
(f) After random period of time the whole system goes for preventive maintenance.
(g) The failure rates of all the units are taken to be exponential whereas the repair time distributions are particular.

Symbols and Notations

E_0 =State of the system at epoch t=0

E = set of regenerative states $S_0 - S_6$

$q_{ij}(t) =$ Probability density function of transition time from S_i to S_j

$Q_{ij}(t) =$ Cumulative distribution function of time to transition time from S_i to S_j

$\pi_i(t) =$ Cumulative distribution function of time to system failure when starting from $E_0 = S_i \in E$ state

$\mu_i(t) =$ Mean Sojourn time in the state $E_0 = S_i \in E$

$B_i(t) =$ Repairman is busy in the repair at time $t / E_0 = S_i \in E$

$r_1 / r_2 / r_3 =$ Constant repair rate of Main unit / Associate units respectively.

α / β = Failure rate of Main Unit / Associate units respectively.

$g_1(t) / g_2(t) / g_3(t)$ = Probability density function of repair time of Main Unit / Associate units / Shut Down mode respectively.

$G_1(t) / G_2(t)$ = Cumulative distribution function of repair time of Main Unit / Associate units respectively.

a(t) = Probability density function of preventive maintenance .

b(t) = Probability density function of preventive maintenance completion time.

A(t) = Cumulative distribution function of preventive maintenance.

B(t) = Cumulative distribution function of preventive maintenance completion time.

S = Symbol for Laplace -stieltjes transform

C = Symbol for Laplace-convolution

$N_o/N_S/N_r$ = unit under operation / good and non – operative mode / repair state

$P_o/P_{WR}/P_r$ = unit under operation / good and non – operative mode / repair state

P.M = System under preventive maintenance

S.D = System under shutdown

Up states - $S_0 = (N_O, P_O, N_s)$; $S_1 = (N_r, P_O, N_O)$; $S_3 = (N_r, P_{wr}, N_O)$; $S4 = (N_O, P_r, N_s)$

Down states - S_2 = (S.D.); S_5 = (P.M.)

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Using Markovian regenerative process, simple probabilistic considerations yield the following non zero transition probabilities (Figure 1):

$$p_{01} = \int_0^\infty \alpha e^{-Xt} \overline{A}(t)dt = \frac{\alpha}{X}[1 - a^*(X)] \quad (1)$$

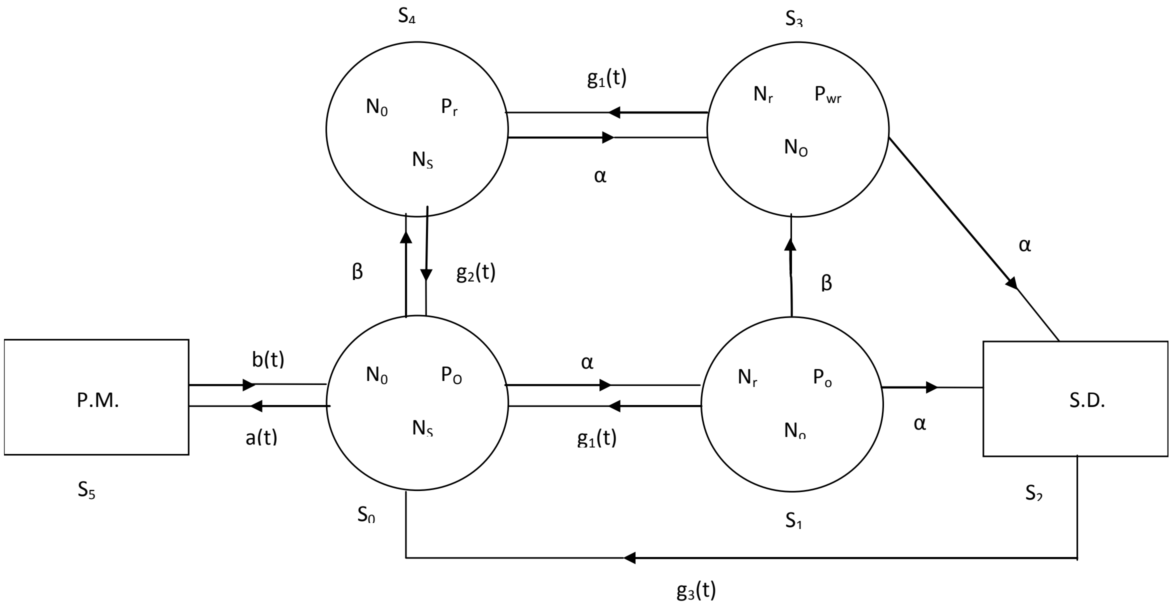

Figure 1. State Transition Diagram.

$$p_{04} = \int_0^\infty \beta e^{-Xt} \overline{A}(t) dt = \frac{\beta}{X}[1 - a^*(X)] \tag{2}$$

$$p_{05} = \int_0^\infty e^{-Xt} a(t) dt = a^*(X) \tag{3}$$

$$p_{10} = \int_0^\infty e^{-Xt} g_1(t) dt = g_1^*(X) \tag{4}$$

$$p_{12} = \int_0^\infty \alpha e^{-Xt} \overline{G_1}(t)\, dt = \frac{\alpha}{X}[1 - g_1^*(X)] \tag{5}$$

$$p_{13} = \int_0^\infty \beta e^{-Xt} \overline{G_1}(t)\, dt = \frac{\beta}{X}[1 - g_1^*(X)] \tag{6}$$

$$p_{20} = \int_0^\infty g_2(t) dt = 1 \tag{7}$$

$$p_{32} = \int_0^\infty \alpha e^{-\alpha t} \overline{G_1}(t)\, dt = 1 - g^*{}_1(X) \tag{8}$$

$$p_{40} = \int_0^\infty e^{-\alpha t} g_2(t)\, dt = g^*{}_2(X) \tag{9}$$

$$p_{43} = \int_0^\infty \alpha e^{-\alpha t} \overline{G_2}(t)\, dt = 1 - g^*{}_2(X) \tag{10}$$

$$p_{50} = \int_0^\infty b(t)\, dt = 1 \tag{11}$$

And the mean sojourn times are given by:

$$\mu_0 = \int_0^\infty e^{-(\alpha+\beta)t} \overline{A}(t) dt = \frac{1}{X}[1 - a^*(X)] \tag{12}$$

$$\mu_1 = \int_0^\infty e^{-(\alpha+\beta)t} \overline{G_1}(t)\, dt = \frac{1}{X}[1 - g_1^*(X)] \tag{13}$$

$$\mu_2 = \int_0^\infty \overline{G_2}(t)\, dt \tag{14}$$

$$\mu_3 = \int_0^\infty e^{-\alpha t} \overline{G_1}(t)\, dt = \frac{1}{X}[1 - g_1^*(X)] \tag{15}$$

$$\mu_4 = \int_0^\infty e^{-\alpha t} \overline{G_2}(t) dt = \frac{1}{X}[1 - g_2^*(X)] \tag{16}$$

$$\mu_5 = \int_0^\infty \overline{B(t)} dt = 1 \tag{17}$$

Where $X = \alpha + \beta$;

Mean time system failure

Let μ_i in the state S_i be defined as the time that system continuous to be in state S_i before transiting to any other states. If T denotes the Sojourn time in state S_i, then time to system failure can be regarded as the first passage time to the failed state. To obtain it we regarded down state as absorbing states. Using argument as for the regenerative process, we obtain the following recursive relation for $\pi_i(t)$ as follows:

$$\mu_i(t) = \int_0^t \pi_i(t-u)dQ_{ij}(u) = \tilde{Q}_{ij}(t) \boxed{S} \pi_i(t)$$

$$\pi_0(t) = \tilde{Q}_{01}(t) \boxed{S} \pi_1(t) + \tilde{Q}_{04}(t) \boxed{S} \pi_4(t) + \tilde{Q}_{05}(t) \quad (18)$$

$$\pi_1(t) = \tilde{Q}_{10}(t) \boxed{S} \pi_0(t) + \tilde{Q}_{13}(t) \boxed{S} \pi_3(t) + \tilde{Q}_{12}(t) \quad (19)$$

$$\pi_3(t) = \tilde{Q}_{34}(t) \boxed{S} \pi_4(t) + \tilde{Q}_{32}(t) \quad (20)$$

$$\pi_4(t) = \tilde{Q}_{04}(t) \boxed{S} \pi_0(t) + \tilde{Q}_{43}(t) \boxed{S} \pi_3(t) \quad (21)$$

Taking Laplace-Stieltje's transform and solving the subsequent matrix

$$\begin{bmatrix} 1 & -\tilde{Q}_{01} & 0 & -\tilde{Q}_{04} \\ -\tilde{Q}_{10} & 1 & -\tilde{Q}_{13} & 0 \\ 0 & 0 & 1 & -\tilde{Q}_{34} \\ -\tilde{Q}_{40} & 0 & -\tilde{Q}_{43} & 1 \end{bmatrix} \begin{bmatrix} \tilde{\pi}_0 \\ \tilde{\pi}_1 \\ \tilde{\pi}_3 \\ \tilde{\pi}_4 \end{bmatrix} = \begin{bmatrix} \tilde{Q}_{05} \\ \tilde{Q}_{12} \\ \tilde{Q}_{32} \\ 0 \end{bmatrix}$$

We get

$$MTSF = \frac{D_1'(0) - N_1'(0)}{D_1(0)}$$

$$MTSF = \frac{((1-p_{34}p_{43})(\mu_0 + \mu_1 p_{01}) + \mu_3(p_{01}p_{13} + p_{04}p_{43}) + \mu_4(p_{01}p_{13}p_{34} + p_{04})}{(1-p_{01}p_{10})(1-p_{34}p_{43}) - p_{40}(p_{01}p_{13}p_{34} + p_{04})} \quad (22)$$

Availability analysis

Let $M_i(t)$ denote the probability that the system is up initially in regenerative state S_i at epoch *t* without passing through any other regenerative state. It might return to itself through one or more non regenerative states so that either it continues to remain in regenerative state without visiting any regenerative state including itself by probability arguments. We observe that the entry to any of the state S_0, S_1, S_2 *and* S_8 is a regenerative point. $A_i(t)$ is defined as the probability that the system is up in state S_0, S_1, S_2 *and* S_8 at epoch *t*.

To obtain it, all possible consequences are considered:

(1) Probability that the system initially up is S_0 is up at epoch *t* without transiting to any other regenerative state in E which is $M_0(t)$.

(2) Probability that the system transits to S_i in E during (u,u+du) and then starting from S_0 it is up at epoch *t* which is

$$A_0 = \int_0^t q_{0i}(t)A_i(t-u)du = q_{0i} \boxed{C} A_i(t) \quad (22)$$

Thus we have

$$A_0(t) = M_0(t) + q_{01}(t) \boxed{C} A_1(t) + q_{04}(t) \boxed{C} A_4(t) + q_{05}(t) \boxed{C} A_5(t) \quad (23)$$

$$A_1(t) = M_1(t) + q_{10}(t) \boxed{C} A_0(t) + q_{12}(t) \boxed{C} A_2(t) + q_{13}(t) \boxed{C} A_3(t) \quad (24)$$

$$A_2(t) = q_{20}(t) \boxed{C} A_0(t) \quad (25)$$

$$A_3(t) = M_3(t) + q_{32}(t) \boxed{C} A_2(t) + q_{34}(t) \boxed{C} A_4(t) \quad (26)$$

$$A_4(t) = M_4(t) + q_{40}(t) \boxed{C} A_0(t) + q_{43}(t) \boxed{C} A_3(t) \quad (27)$$

$$A_5(t) = q_{50}(t) \boxed{C} A_0(t) \quad (28)$$

Taking Laplace-transform of the above equations and writing in matrix form:

$$\begin{bmatrix} 1 & -q_{01}^* & 0 & 0 & -q_{04}^* & -q_{05}^* \\ -q_{10}^* & 1 & -q_{12}^* & -q_{13}^* & 0 & 0 \\ -q_{20}^* & 0 \quad 1 & 0 & 0 & 0 & \\ 0 & 0 & -q_{32}^* & 1 & -q_{34}^* & 0 \\ -q_{40}^* & 0 & 0 & -q_{43}^* & 1 & 0 \\ -q_{50}^* & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_0^* \\ A_1^* \\ A_2^* \\ A_3^* \\ A_4^* \\ A_5^* \end{bmatrix} = \begin{bmatrix} M_0^* \\ M_1^* \\ 0 \\ M_3^* \\ M_4^* \\ 0 \end{bmatrix}$$

We get the following expression for Availability:

$$A_0(\infty)=\frac{\mu_0(1-p_{34}p_{43})+\mu_1 p_{01}(1-p_{34}p_{43})+\mu_3(p_{01}p_{13}-p_{04}p_{43})+\mu_4(p_{04}+p_{01}p_{13}p_{34})}{(1-p_{34}p_{43})[\mu_o+\mu_1 p_{01}+\mu_5 p_{05}]+\mu_2[p_{01}p_{12}(1-p_{34}p_{43})+p_{32}(p_{01}p_{13}+p_{04}p_{43})]+\mu_3(p_{01}p_{13}+p_{04}p_{43})+\mu_4(p_{04}+p_{01}p_{13}p_{34})} \quad (29)$$

BUSY PERIOD ANALYSIS

Busy period repairman for performing normal repair

Let $W_i(t)$ denote the probability that the repairman is busy initially with repair in regenerative state S_4 and remains busy at epoch *t* without transiting to any other state or returning to itself through one or more regenerative states. By probabilistic argument, we have:

$$W_i(t)=\overline{G_i}(t)$$

Developing similar relationship as in availability for normal repair, we have to find the steady state, the fraction of time for which the repairman of busy with normal repair is given by:

$$B_0^{1}(\infty)=\lim_{s\to 0} B_0^{*1}(t)=\frac{N_3(0)}{D_2^{'}(0)}$$

$$N_3(0)=\mu_1 p_{01}(1-p_{34}p_{43})+\mu_3(p_{01}p_{13}+p_{04}p_{43})-\mu_4(p_{14}+p_{01}p_{13}p_{34}) \quad (30)$$

Where $D_2^{/}(0)$ is same as in Availability expression.

Therefore, in the long run, the fraction of time for the repairman in busy with the normal repair is given by:

$$B_0^{1}(\infty)=\frac{\mu_1 p_{01}(1-p_{34}p_{43})+\mu_3(p_{01}p_{13}+p_{04}p_{43})-\mu_4(p_{14}+p_{01}p_{13}p_{34})}{(1-p_{34}p_{43})-p_{01}p_{10}(1-p_{34}p_{43})-p_{05}p_{50}(1-p_{34}p_{43})-p_{20}p_{01}p_{12}(1-p_{34}p_{43})-p_{01}p_{13}p_{32}p_{20}-p_{04}p_{43}p_{32}p_{20}-p_{40}p_{04}-p_{01}p_{13}p_{34}p_{40}} \quad (31)$$

Busy period repairman performing for shutdown repair

Similarly, to find the steady state the fraction of time for which the repairman of busy with shut down repair:

$$B_0^{2}(\infty)=\frac{N_4(0)}{D_2^{'}(0)}=\frac{\mu_2[p_{01}p_{12}(1-p_{34}p_{43})+p_{32}(p_{01}p_{13}+p_{04}p_{43})]}{(1-p_{34}p_{43})-p_{01}p_{10}(1-p_{34}p_{43})-p_{05}p_{50}(1-p_{34}p_{43})-p_{20}p_{01}p_{12}(1-p_{34}p_{43})-p_{01}p_{13}p_{32}p_{20}-p_{04}p_{43}p_{32}p_{20}-p_{40}p_{04}-p_{01}p_{13}p_{34}p_{40}} \quad (32)$$

Busy period of repairman performing the preventive maintenance

Similarly, in the long run, for the fraction of time the repairman in busy with the preventive maintenance is given by:

$$B_0^{3}(\infty)=\frac{N_5(0)}{D_2^{'}(0)}=\frac{\mu_5 p_{05}(1-p_{34}p_{43})}{(1-p_{34}p_{43})[\mu_o+\mu_1 p_{01}+\mu_5 p_{05}]+\mu_2[p_{01}p_{12}(1-p_{34}p_{43})+p_{32}(p_{01}p_{13}+p_{04}p_{43})]+\mu_3(p_{01}p_{13}+p_{04}p_{43})+\mu_4(p_{04}+p_{01}p_{13}p_{34})}$$

Particular case

Case (i)

A random variable is said to have the Weibull distribution if its distribution is given, for some λ>0 ,α>0 by

$$G(t)=1-e^{-(\lambda t)\alpha},\ t\geq 0$$

The failure rate function for Weibull distribution equals

$$\lambda(t)=\frac{e^{-(\lambda t)^{\alpha}}\alpha(\lambda t)^{\alpha-1}\lambda}{e^{-(\lambda t)^{\alpha}}}=\alpha\lambda(\lambda t)^{\alpha-1}$$

Thus, the Weibull distribution is IFR (Increasing Failure Rate) when α≥0 and DFR (Decreasing Failure Rate) when 0<α≤1. When α=1, $G(t)=1-e^{-(\lambda t)\alpha}$, the exponential distribution which is both IFR and DFR.

We have

$$p_{12}=\frac{\alpha}{X+r_1};\ p_{13}=\frac{\beta}{X+r_1};\ p_{20}=\frac{\alpha}{X+\theta};\ p_{16}=\frac{\theta}{X+\theta};\ p_{21}=\frac{r_2}{Z+r_2}$$

$$p_{01}=\frac{\alpha}{X+\theta};\ p_{04}=\frac{\beta}{X+\beta};\ p_{05}=\frac{\theta}{X+\theta};\ p_{10}=\frac{r_1}{X+r_1}$$

$$p_{40}=\frac{r_2}{\alpha+r_2};\quad p_{43}=\frac{\alpha}{Y+r_2};$$

and

$$\mu_0=\frac{1}{X+\theta};\ \mu_1=\frac{1}{X+r_1};\ \mu_2=\frac{1}{r_2};\ \mu_3=\frac{1}{\alpha+r_1}$$

$$\mu_4=\frac{1}{\alpha+r_2};\ \mu_5=\frac{1}{\eta};$$

Then we have,

$$MTSF=\frac{L_0L_1M_0+L_3+L_2L_4}{M_1L_1-L_2r_2L_4} \quad (34)$$

$$Availability=\frac{L_1L_2+L_3+K_1L_4}{L_0L_1(M_0+N_0)+\alpha N_1(L_0L_1N_2+L_3)+(L_3+L_2L_4)}; \quad (35)$$

Busy period

$$B_0^{1*}(\infty)=\frac{L_0L_1N_2+L_3-L_2L_4}{L_0L_1(M_0+N_0)+\alpha N_1(L_0L_1N_2+L_3)+(L_3+L_2L_4)}; \quad (36)$$

$$B_0^{2*}(\infty)=\frac{\alpha N_2[L_0N_2(1+L_1)+L_2N_3}{L_0L_1(M_0+N_0)+\alpha N_1(L_0L_1N_2+L_3)+(L_3+L_2L_4)}\;; \quad (37)$$

$$B_0^{3*}(\infty)=\frac{L_0L_1N_0}{L_0L_1(M_0+N_0)+\alpha N_1(L_0L_1N_2+L_3)+(L_3+L_2L_4)} \quad (38)$$

Where

$$L_0=\frac{1}{X+\theta}\;;\; L_1=1-\frac{\alpha r_1}{(\alpha+r_1)(\alpha+r_2)}\;;\; L_2=\frac{\beta}{(X+\theta)(\alpha+r_2)};$$

$$L_3=\frac{\alpha\beta}{(X+\theta)(\alpha+r_1)}\left[\frac{1}{\alpha+r_2}+\frac{1}{X+r_1}\right];\; L_4=1+\frac{\alpha r_1}{(X+r_1)(\alpha+r_1)}\;;$$

$$M_0=1+\frac{\alpha}{X+r_1};L\backslash M_1=1-\frac{\alpha r_1}{(X+\theta)(X+r_1)}\;;$$

$$N_0=\frac{\theta}{\eta}\;;\; N_1=\frac{1}{r_2}\;;\; N_2=\frac{\alpha}{X+r_1}\;; N_3=\frac{\alpha}{\alpha+r_1}\;;$$

Case (ii)

When all repair time distribution are n-phases Erlangian distribution, that is, Density Function and Survival Function

$$g_i(t)=\frac{nr_i(nr_it)^{n-1}e^{-nr_it}}{n-1!}\quad;\quad \overline{G}_i(t)=\sum_{j-0}^{n-1}\frac{(nr_it)^j e^{-nr_it}}{j!}\;;$$

And other distribution are negative exponential

$$a(t)=\theta e^{-\theta t}\;,\; b(t)=\eta e^{-\eta t}\;,\; \overline{A}(t)=e^{-\theta t}\;,\; \overline{B}(t)=e^{-\eta t}$$

Taking n=2, we get

$$g_i(t)=\frac{2r_i(2r_it)^1 e^{-2r_it}}{1!}$$

$$g_1(t)=4r_1^{\,2}\,te^{-r_1t}\;;\quad g_2(t)=4r_2^2te^{-2r_2t}\;;\; g_3(t)=4r_3^2te^{-2r_3t}$$

$$\overline{G}_1(t)=\sum_{j=0}^{1}\frac{(2r_1t)^j e^{-r_1t}}{j!}$$

$$\overline{G}_1(t)=e^{-2r_1t}+(2r_1t)e^{-2r_1t}\;;\qquad \overline{G_2}(t)=e^{-2r_2t}+(2r_2t)e^{-2r_2t}$$

$$p_{01}=\frac{\alpha}{X_1}\;;\; p_{04}=\frac{\beta}{X_2}\;;\; p_{05}=\frac{\theta}{X_1}\;;\; p_{10}=\frac{4r_1^2}{X_2^{\,2}}\;;\; p_{12}=\frac{\alpha}{X_2}+\frac{2\alpha r_1}{X_2^{\,2}}\;;\; p_{13}=\frac{\beta}{X_2}+\frac{2\beta r_1}{X_2^{\,2}}\;;\; p_{20}=1\;;$$

$$p_{32}=\frac{X_4}{X_3^{\,2}}\;;\; p_{34}=\frac{4r_1^2}{X_3^{\,2}}\;;$$

$$p_{40}=\frac{4r_2^{\,2}}{X_6^{\,2}}\;;\; p_{43}=\frac{X_5}{X_6^{\,2}}\;;\; p_{50}=1;$$

and

$$\mu_0=\frac{1}{X_1}\;;\; \mu_1=\frac{X_2+2r_1}{X_2^{\,2}}\;;\; \mu_2=\frac{1}{r_2}$$

$$\mu_3=\frac{4r_1^2}{X_3^{\,2}}\;;\mu_4=\frac{\alpha+4r_2}{X_6^{\,2}}\;;\; \mu_5=\frac{1}{\eta}$$

where

$$X_1=\alpha+\beta+\theta;\qquad X_2=\alpha+\beta+2r_1;\qquad X_3=\alpha+2r_1;$$

$$X_4=\alpha^2+4\alpha r_1;\qquad X_5=\alpha^2+4\alpha r_2;\qquad X_6=\alpha+2r_2$$

$$MTSF=\frac{(1-\frac{4r_1^2X_5}{X_3^{\,2}X_6^{\,2}})[\frac{X_2^{\,2}+\alpha X_2+2\alpha r_1}{X_1X_2^{\,2}}]+\frac{4r_1^2}{X_3^{\,2}}\{\frac{\beta X_5}{X_1X_6^{\,2}}+\frac{\alpha\beta}{X_1X_2}(1+\frac{2r_1}{X_2})\}+[\frac{\beta}{X_1}+\frac{4\alpha r_1^2}{X_1X_3^{\,2}}(\frac{\beta}{X_2}+\frac{2\beta r_1}{X_2^{\,2}})]}{(1-\frac{4\alpha r_1^2}{X_1X_2^{\,2}})(1-\frac{4r_1^2X_5}{X_3^{\,2}X_6^{\,2}})-\frac{4r_2^2}{X_6^{\,2}}[\frac{\beta}{X_1}+\frac{4\alpha r_1^2}{X_1X_3^{\,2}}(\frac{\beta}{X_2}+\frac{2\beta r_1}{X_2^{\,2}})]} \quad (39)$$

Availability = $\dfrac{N_2(0)}{D_2^{/}(0)}$

Where

$$N_2(0)=(1-\frac{4r_1^2X_5}{X_3^2X_6^2})(\frac{X_2^2+\alpha X_2+2\alpha r_1}{X_1X_2^2})+\frac{4r_1^2\beta}{X_1(\alpha+2r_1)^2}[\frac{\alpha}{X_2^2}(X_2+2r_1)+\frac{X_5}{X_6^2}]+ +\frac{4r_2^2}{X_1X_6^2}[1+\frac{4\alpha r_1^2}{X_2X_3^2}(1+\frac{2r_1}{X_2})] \quad (40)$$

$$D_2^{/}(0)=\frac{1}{X_1}(1-\frac{4r_1^2X_5}{X_3^2X_6^2})[\frac{1}{\eta}+\frac{X_2^2+\alpha X_2+2\alpha r_1}{X_2^2}]+\frac{1}{r_2}[\frac{\alpha^2}{X_1X_2}(1+\frac{2r_1}{X_2})(1-\frac{4r_1^2X_5}{X_3^2X_6^2}) +\frac{\beta X_4}{X_1X_3^2}[\frac{X_5}{X_6^2}+\frac{\alpha}{X_2}(1+\frac{2r_1}{X_2})]+\frac{4\beta r_1^{2^{br_1^2}}}{X_1(\alpha+2r_1)^2}[\frac{\alpha}{X_2^2}(X_2+2r_1)+\frac{X_5}{X_6^2}] +\frac{4\beta r_2^2}{X_1X_6^2}[1+\frac{4\alpha r_1^2}{X_2X_3^2}(1+\frac{2r_1}{X_2})] \quad (41)$$

Busy period analysis:

(i) $B_0^1(\infty)=\dfrac{N_3(0)}{D_2^{/}(0)}$ (ii) $B_0^2(\infty)=\dfrac{N_4(0)}{D_2^{/}(0)}$ (iii) $B_0^3(\infty)=\dfrac{N_5(0)}{D_2^{/}(0)}$

Where

$$N_3(0)=\frac{\alpha(X_2+2r_1)}{X_1X_2^2}[(1-\frac{4r_1^2X_5}{X_3^2X_6^2})+\frac{4r_1^2\beta}{X_1(\alpha+2r_1)^2}[\frac{\alpha}{X_2^2}(X_2+2r_1)+\frac{X_5}{X_6^2}] -\frac{4r_2^2}{X_1X_6^2}[1+\frac{4\alpha r_1^2}{X_2X_3^2}(1+\frac{2r_1}{X_2})] \quad (42)$$

$$N_4(0)=\frac{1}{r_2}[\frac{\alpha^2}{X_1X_2}((1+\frac{2r_1}{X_2})(1-\frac{4r_1^2X_5}{X_3^2X_6^2})+\frac{\beta X_4}{X_1X_3^2}(1+\frac{2r_1}{X_2})+\frac{\beta X_4X_5}{X_1X_3^2X_6^2}] \quad (43)$$

$$N_5(0)=\frac{\theta}{\eta X_1}(1-\frac{4r_1^2X_5}{X_3^2X_6^2}) \quad (44)$$

Profit analysis

The profit analysis of the system can be carried out by considering the expected busy period of repairman in repair of the unit in [0,t]. Therefore, G(t)= total revenue earned by the system in [0,t]- Expected repair cost in [0,t]

$$=C_1\mu_{up}(t)-C_2\mu_{b1}-C_3\mu_{b2}-C_4\mu_{b3} \quad (45)$$

where

$$\mu_{up}(t)=\int_0^t A_0(t)dt \quad ; \quad \mu_{b1}(t)=\int_0^t B_0^1(t)dt \quad ; \quad \mu_{b2}(t)=\int_0^t B_0^2(t)dt \quad ; \mu_{b3}(t)=\int_0^t B_0^3(t)dt$$

RESULTS AND DISCUSSION

Case I: When distribution is taken to be Weibull as shown in Tables 1 to 3.

Case II: When distribution is taken to be Erlangian as shown in Tables 4 to 6.

An excellent work in this direction involving components of two-unit system was done by Gaver (1964) and Harris (1968) but the comparative study of various parameters was not taken into account by any of these authors. We have considered two distributions namely Erlangian and Weibull which are regarded as the best distributions for achieving optimum results. We have employed regenerative point technique for obtaining mean time to system failure, availability and busy period analysis which are helpful in performing the profit analysis for arbitrary repair time distribution.

However, the whole work could also have been viewed with the help of developing differential equations and taking Laplace-Transform thereof and Inverse Laplace-Transform after that and reliability analysis could also have been performed as well as performance evaluation could have been undertaken, which the authors plan to carry out in the next work.

Conclusion

It is observed that the mean time to system failure (MTSF) and availability of the system decreases rapidly with the increase of failure rates α & β for fixed values of

Table 1.Variation in MTSF vis-a-vis failure rate of main unit.

α , β	η,θ	r_3	r_1, r_2	MTSF
0.01	0.005	0.1	0.01	255.923
0.02	0.005	0.1	0.02	108.462
0.03	0.005	0.1	0.03	102.308
0.04	0.005	0.1	0.04	92.254

Table 2. Variation in Availability vis-a-vis failure rate of main unit.

α , β	η,θ	r_3	r_1, r_2	Availability
0.01	0.005	0.1	0.01	241.252
0.02	0.005	0.1	0.02	124.977
0.03	0.005	0.1	0.03	114.666
0.04	0.005	0.1	0.04	100.358

Table 3. Variation in Profit vis-a-vis increase in failure rate of main unit.

α , β	η,θ	r_3	r_1, r_2	Profit
0.01	0.005	0.1	0.01	146.259
0.02	0.005	0.1	0.02	144.122
0.03	0.005	0.1	0.03	122.969
0.04	0.005	0.1	0.04	111.027

Table 4. Variation in MTSF vis-a-vis failure rate of main unit.

α , β	η,θ	r_3	r_1, r_2	MTSF
0.01	0.005	0.1	0.01	324.675
0.02	0.005	0.1	0.02	310.491
0.03	0.005	0.1	0.03	310.230
0.04	0.005	0.1	0.04	309.922

Table 5. Variation in Availability vis-a-vis failure rate of main unit.

α , β	η,θ	r_3	r_1, r_2	Availability
0.01	0.005	0.1	0.01	341.933
0.02	0.005	0.1	0.02	340.127
0.03	0.005	0.1	0.03	340.116
0.04	0.005	0.1	0.04	330.003

Table 6. Variation in Profit vis-a-vis failure rate of main unit.

α , β	η,θ	r_3	r_1, r_2	Profit
0.01	0.005	0.1	0.01	306.211
0.02	0.005	0.1	0.02	284.312
0.03	0.005	0.1	0.03	262.299
0.04	0.005	0.1	0.04	211.033

other parameters, when the distribution is taken to be Weibull. However, it is noted that when the distribution is assumed to be n-phase Erlangian, the mean time to system failure and availability of the system do not decrease so rapidly. Same can be predicted for profit analysis also.

Conflict of Interest

The author(s) have not declared any conflict of interest.

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